

# Thinking with Networks

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## Abstract

Many planning tasks involve complex reasoning about time: what must happen in sequence and what may happen in parallel. One hundred ten online participants were provided with a simple planning scenario (to design a calling tree) and asked to manipulate different diagrammatic representations of the problem. More important than the initial representation was the participants' transformed representations: if time was encoded in the lengths of tree links then inference was more accurate. This finding suggests that diagram transformation may be a useful way to elicit representation strategies, and that such transformations from different starting conditions may be useful as diagnostics and as design aids.

**Keywords:** diagram understanding, design, topological diagrams, representation of time, distributed computing

## Introduction

To solve problems or to simply organize information, people often make diagrams. Diagrams can aid problem solving and information organization by spatializing the essential concepts and relations among them. One of the most abstract kinds of diagrams is a network, where nodes are concepts and links are relations. Because of their generality, network diagrams appear in many diverse domains.

The advantage of networks, their ability to represent so many different relations, is also a disadvantage, because they may not make problem constraints apparent. Often, problems and information have more constraints than the

simple binary relations used in networks, constraints that would allow inferences, for example, asymmetric relations. Certain variants of networks can represent such constraints. Trees, for example, are commonly used to represent asymmetric or hierarchical relations, notably for structural relations such as organization charts or phylogenetic relations. They are also used to represent temporal relations, as in decision trees or flow diagrams. For structure, the links indicate an asymmetric structural relation, such as control in corporations or *kind of* in phylogenies. For time, the links indicate asymmetric temporal relations, at an ordinal level: this, then this, then this.

However, there are situations where representing both structural and temporal relations is desired, for example, in coordination situations where a set of agents carry out a temporally constrained set of actions. Representing both structural and temporal organizations simultaneously presents a challenge. The structure – who contacts whom – needs to be represented. Also the timing – the temporal ordering of contacts – must be shown. Links can be used for the structural information, but some other aspect of the diagram needs to be used for the temporal. Representing both structure and time simultaneously can be all the more challenging when *metric* properties of time are important because links in networks are typically used to indicate a relationship, but not the degree of a relationship.

These problems of representation are, more broadly, problems of cognition: as the data will show, reasoning

about coordination is difficult. Given the high cost of coordination failure in a number of different fields, and the everyday importance of coordination in computational fields, the problem deserves attention. Previously, studies have been performed on the way network diagrams convey information. For example, it has been shown that distance along network links is used to evaluate content similarity (Fabrikant, Montello, Ruocco, & Middleton, 2004). That study focused on the way distance and topology map to similarity; here we focus on the way distance and topology map to structure and time.

We turned to users to see how they would represent space and time simultaneously. Often, users turn out to be good designers, inventing clever devices to represent abstract information (e. g., Kessell and Tversky, 2008; Tversky, in press). Furthermore, their visualizations of thought are a window to thought (e. g., Tversky and Lee, 1999). The other side of successful design is comprehension. We have begun exploring how people design and comprehend diagrams and solutions for a class of problems that requires representing both structure and time (see Figure 1).

The paradigm we have been using is based on a distribution tree. Because the problem is a general one of transmission of something from one party to many, it applies to many situations when information or goods are distributed. For example, a telephone tree can be used to distribute information about a school closing due to weather conditions. For speed of transmission, it is better to distribute the callers; for reliability, it is better to minimize the number of callers. Solutions, then, depend both on structure and on time. Although some forms of trees, such as decision trees and flow diagrams, are used to represent time, they only represent temporal *order*. In contrast, optimizing a distribution tree depends on *metric* properties of time as well. Thus, diagramming a distribution tree solution not only requires representing both structure and time, it also requires representing time metrically. An added difficulty for designers and for users in producing or interpreting designs for distribution trees is that several calls can happen at the same time. That is, both sequence and parallelism need to be represented and understood.

In extensive pilot work, we have found that people spontaneously create trees to solve these problems, but that their trees usually represent structure, that is, who contacts whom, and rarely represent time. In fact, representing or grasping structure from diagrams is easier and more straight-forward than representing or grasping changes in structure, such as changes in time (e. g., Suwa & Tversky, 1997; Tversky, Heiser, Lozano, Mackenzie, & Morrison, 2008). More generally, space seems to serve as a metaphor for time more readily than time for space (Boroditsky, 2000).

For the telephone tree problem, structure is ordinal, but time is metric. There are several ways to superimpose time onto a network representing structure. Telephone trees are tricky because a single agent can make only one call at a time, but several agents can call simultaneously. One way to

represent time, illustrated in Figure 2, is to use length of link, as in additive similarity trees (Sattath & A. Tversky, 1977; Corter, 1996). In this representation, the lengths of the links emanating from any one agent indicate the sequence of that caller's calls. For large trees, this can be visually confusing. Another method to represent time is a combination of using levels of a tree to distinguish when a caller is first notified, and within levels, showing the sequence of calls made by a caller using a left-to-right first-to-last convention; this is visually more organized but requires keeping track of two spatial mappings to assess time. Both methods have been invented by our participants. Here, we investigate solution success when time is or is not represented by length of line.

As noted, users can be effective designers of visualizations of problems. Does the very process of designing visualizations facilitate using them? Architects and other designers sketch designs, study their sketches, get new insights, and revise them, a positive, productive cycle that has been likened to a conversation (Schon, 1983). Creating and revising visualizations of a range of complex concepts, for example, scientific ones, has been shown to increase depth of understanding (e. g., diSessa, Hammer, Sherin, & Kolpakowski, 1991; Schwartz, 1995).

The present experiment examines the dual roles of kind of design and act of designing for solving telephone tree problems. Participants were given a problem analogous to the guard problem and then asked to provide an optimal diagram and to compute the amount of time it should take to notify everyone. They were given an initial diagram, one they could alter, to create a diagram they regarded as optimal. For some participants, the initial diagram represented structure but not time. For others, the initial diagram represented time using proportional length of line. For a third group, the initial diagram varied in line length but not proportionally to time; thus this diagram provides a hint that line length might be helpful, but not how.

This design allows asking a set of questions. We can ask whether representing time explicitly in a visualization makes for a more effective diagram that better helps users to solve a telephone tree problem. We can ask whether time is more likely to be explicitly represented in user diagrams when the starting diagram provided to them uses variable line lengths, either compatible with time or incompatible with time.

## Method

One hundred ten participants accepted and completed an assigned task in return for payment on Amazon's crowdsourcing marketplace. The participants in Amazon's pool have been characterized extensively in several previous studies: the pool is 55% female with a mean age of 31 (Kittur, Chi, & Su, 2008; Ross, Irani, Silberman, Zaldivar, & Tomlinson, 2010). Participants were presented with the following textual description:

Please read the following question and then make changes to the diagram.

Hart has the job of notifying 4 other parents in the event that school is called off due to weather conditions. Hart has created a plan for a sequence of phone calls:

**Hart calls Dean and then Lane. Dean calls Boyd and then Ward.**

Assuming that each phone call lasts one minute, please go to the website below to make changes to the diagram to meet the plan description.

Once they had saved the diagram, they were asked:

Assuming that each call lasts one minute, how many minutes will elapse before all parents know about the school cancellation?

Participants were randomly assigned one of the following three diagrams shown in Figures 1, 2 and 3. Figure 1 is a typical tree structure. The connections indicate who calls whom, using uniform line lengths. In a pilot study, most participants drew such a diagram.

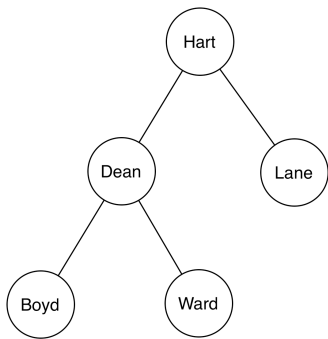


Figure 1: A uniform tree with no time encoding

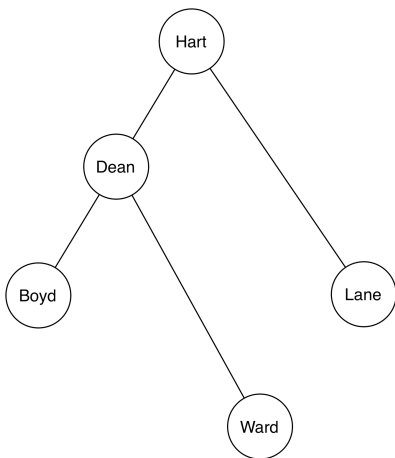


Figure 2: A time-encoded tree with edge lengths consistent with the problem description

In Figure 2, time has been encoded into the lengths of the connections between nodes. That is, after one minute, Hart has managed to talk to Dean. After two minutes, Hart has also managed to talk to Lane, and Dean has talked to Boyd.

The lengths of the connections reflect the constraint of the problem, that a person can only have one phone conversation, and so time has elapsed after each conversation. We found in a paper and pencil pilot study that some participants invented or at least used this representation. It is similar to diagrams used in transportation systems called space-time networks, in which nodes are lined up according to elapsed time (e.g., Pallottino & Scutella, 1998).

In Figure 3, variable edge lengths are used, but the vertical position of a node is not consistent with elapsed time in the problem. For example, in the problem statement, Dean calls Boyd before calling Ward, and the vertical arrangement of Figure 3 implies the opposite order. Thus, the diagram may cue individuals to the possibility of using connection length to represent time, but is not useful in representing the problem (and may even be misleading) unless it is transformed.

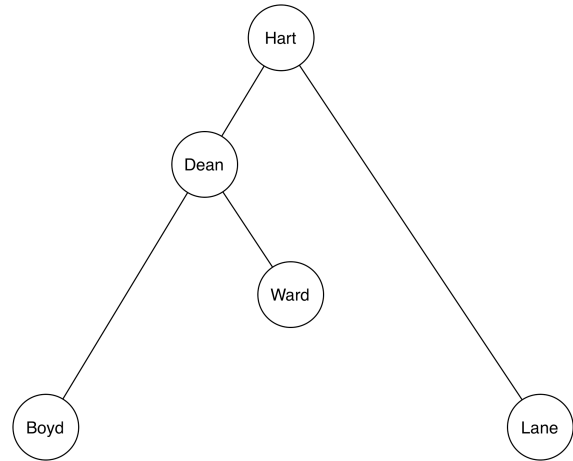


Figure 3: A time-encoded tree with edges inconsistent with the problem description

After being randomly assigned one of the three diagrams above, participants were provided instructions on how to use a customized web-delivered vector-based drawing tool to move nodes, thereby manipulating spacing in the diagram. In the tool, the connections between the nodes are preserved as the nodes are moved. The participants' mouse movements were recorded. Thus, the experiment allows us to study the effect of the initial diagram provided, the cuing representation. In addition, the participants' transformed diagrams can be classified, and the relationship between these produced diagrams and the accuracy of problem solving shown.

## Results

A total of 32 participants were cued with the uniform tree of Figure 1, 38 with the consistent time tree of Figure 2, and 40 with the inconsistent time tree of Figure 3. The overall accuracy of their answer to the time question as a function of cuing diagram is shown in Figure 4 as a proportion. The consistent time tree was associated with the highest accuracy (.66), and the inconsistent time tree with lowest accuracy (.48); the uniform tree produced an intermediate level of accuracy (.53). In a logistic regression model comparing accuracy for these three conditions, cuing with the time tree yielded marginally higher accuracy than the uniform tree (Wald = 2.618,  $.05 < p < .10$ , one-tailed) and cuing with an inconsistent time tree produced marginally lower accuracy than the other two trees (Wald = 1.983,  $.05 < p < .10$ , one-tailed).

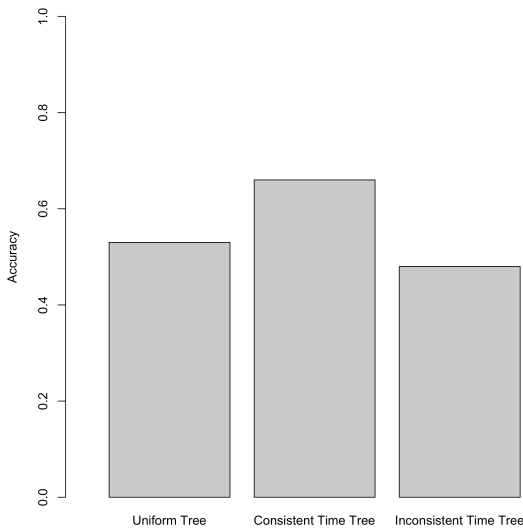


Figure 4: Accuracy of the answer depending upon the randomly provided starting diagram

The final tree diagrams produced by the participants were then classified into three sets: Uniform Trees, Time Trees, and Wrong Trees. Wrong trees could only result if participants used the drawing tool to change the topology of the graphs by adding or subtracting nodes. For example, one participant directly linked Dean to Lane, as in Figure 5.

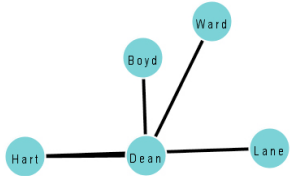


Figure 5: A topologically incorrect tree (Wrong Tree)

The rest of the participants created two kinds of topologically valid trees. Those producing Uniform Trees

showed no attempt to encode time through distance, while those producing Time Trees did. Classifying the trees was straightforward, because the uniform trees tended to have uniform distances, and in particular equal distances between parents and direct descendants. We checked inter-rater reliability of the coding of the produced graphs by training two raters on 15 graphs and then testing on 43 graphs: Cronbach's alpha = .99. Figure 6 shows examples of the produced trees.

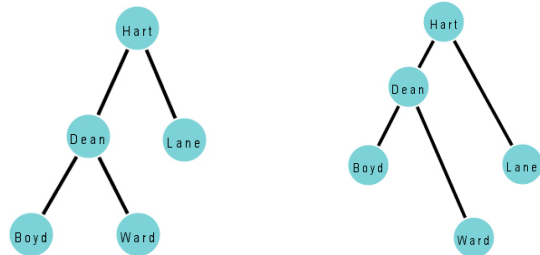


Figure 6: On the left, a produced uniform tree transformed from a given consistent time tree, and on the right, a consistent time tree transformed from a uniform tree.

Figure 7 shows accuracy, as a proportion of participants' time estimates by type of produced diagrams. For example, the left-most blue and tan bars show that 73% of those who produced time trees when provided with uniform trees calculated the correct answer, whereas 50% of those who produced a uniform tree in that same condition calculated the correct answer.

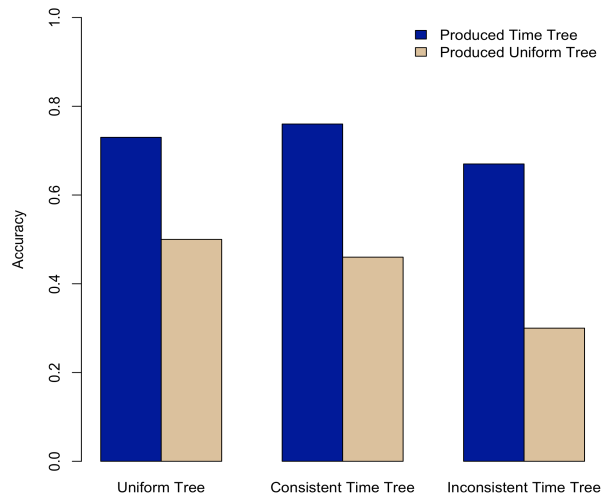


Figure 7: Accuracy of the answer depending upon the diagram produced by the participant, grouped by the starting diagram.

The number of participants in each category can be found in Table 1, which lists the accuracy for each category, the number of participants in each category, and the totals. The

accuracy of those who produced Time Trees was significantly greater than those who produced Uniform Trees,  $\chi^2(1) = 7.58, p < .01$ .

Table 1: Mean accuracy (and frequency) for combinations of given and produced trees

		Starting diagram			
		Uniform Tree	Consistent Time Tree	Inconsistent Time Tree	Overall (Total)
Produced diagram	Uniform Tree	.50 (18)	.46 (13)	.29 (17)	.42 (48)
	Consistent Time Tree	.73 (11)	.76 (21)	.85 (13)	.78 (45)
	Inconsistent Time Tree	-- (0)	-- (0)	.00 (4)	.00 (4)
	Wrong Tree	.00 (3)	.75 (4)	.50 (6)	.46 (13)
	Overall (Total)	.53 (32)	.66 (38)	.48 (40)	

Even when presented with a consistent time tree, six participants altered the tree into a uniform tree. That is, some participants went out of their way to reconfigure the most effective diagram type to a simpler type. On the other hand, eleven participants changed the inconsistent time tree to a consistent time tree, and these participants achieved the highest accuracy shown in the table: 85% got the problem right.

Starting from the inconsistent time tree condition, four subjects produced inconsistent time trees. An example is shown in Figure 8: The tree is inconsistent because Boyd is called before Ward, yet Boyd is placed farther away from Dean than is Ward. These inconsistent trees occurred in no other condition. More broadly, from an examination of the drawing logs, we found that participants will sometimes just modify a diagram slightly, as opposed to drastically or not at all.

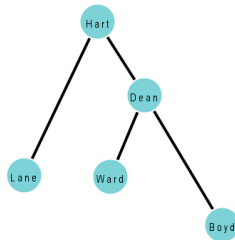


Figure 8: An inconsistent time tree, transformed from an inconsistent time tree.

## Discussion

One hundred and ten participants were asked to diagram and solve a telephone tree problem, that is, determine the structure of a call tree that would notify everyone the fastest, and then to use the call tree to compute the total time to call everyone. The implicit challenge was to design a diagram that simultaneously represented both the structure of the telephone tree and the time to accomplish the plan. To

assess the effects of cuing, participants were given one of three starting diagrams representing the structure of the plan: one that did not represent time; one that represented time with line lengths proportional to time; one that represented time with line lengths inversely proportional to time. There were two critical questions. Would diagrams that represent time lead to better solutions? Would cuing with a time diagram improve designs and solutions?

Those participants who created a diagram that represented time with line length were far more successful at computing the total time to call all agents than those who produced diagrams that did not represent time. Although diagrams not representing time and even some that represented time in a confusing way could be used to compute the correct solution, explicitly representing time led to large increases in correct solutions. Thus, using a diagram that directly represents all the information needed to compute the answer facilitates computation and performance.

Cuing participants with starting diagrams that did or did not represent time in a compatible way, that is, using line length proportional to time, had effects, if small, on successful solution, mediated by the final diagram participants produced.

The results show that reasoning about parallel and sequential events is difficult. Presented with a simple example involving a small number of nodes, participants in the study failed to infer the total time of a process about half the time. Presenting a diagrammatic representation that encodes time helped some, as did manipulating a diagram into a representation that encoded time.

There are implications for diagram design as well as diagram use. First, people do not always design diagrams that capture all the essential components of a situation or a problem. The present project elucidates one reason for the failure: some features of situations or problems are more readily spatialized than others. Importantly, space and structure, static relations, are more likely to be represented in diagrams and more likely to be interpreted correctly than more abstract features such as time. Representing structure and time simultaneously is especially difficult, all the more so because independently, each would select the same diagrammatic feature, lines linking nodes to nodes. Finding a second diagrammatic feature to represent the second variable, in this case time, is a challenge if only because time is unidimensional, best represented as a single line. Producing the right diagram, just like producing the right mental representation, facilitates problem solving.

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