

Mapping the Common Core State Standards

to Advanced Mathematical Knowledge for Teaching

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Methodology

- First round: Individual coding by 2 researchers
- Second round: Joint coding between researchers
- Coding Framework Development
- Third round: Individual re-coding, according to jointly established framework
- Fourth round: validity check, conflict resolution



Notes: Levels*

- L1 – mathematical awareness
- L2 – understand the heart of an idea; working knowledge of examples and counterexamples
- L3 – rigorous proof level understanding

*we mapped each Standard to the advanced math topic (being as specific as possible) and quickly realized a need to distinguish what level of understanding of that topic would be necessary for teaching the given Standard.

Example: Elementary

- **CCSS-M: 5.OA.3.** Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane.
- **AMK: Calculus of Functions, Sequences & Series – L1.** *Mathematical standards for which teaching would be enhanced by teachers' knowledge of the construction and analysis of finite and infinite sequences and series; including use of Taylor series for approximating non-polynomial functions*

Example: Elementary

- **CCSS-M: 3.OA.9.** Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*
- **AMK: Number Theory, Even/Odd Numbers – L3.** *Mathematical standards for which teaching would be enhanced by teachers' knowledge of the properties for operations with even/odd numbers (e.g, even x odd = even, etc.) and their mathematical justification; also, identification and justification of emerging patterns based on even/odd*

Example: Middle

- **CCSS-M: 8.EE.5.** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.
- **AMK: Calculus of Functions, Rate of Change/ Derivative – L3.** *Mathematical standards for which teaching would be enhanced by teachers' knowledge of average and instantaneous rates of change (distinctions and similarities); also, a generalized notion of derivative when introducing concepts related to slope*

Example: Secondary

- **CCSS-M: F-BF.4** Find inverse functions. Solve an equation of the form $f(x)=c$ for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x)=2x+3$ or $f(x)=x+1/x-1$ for $x \neq 1$
- **AMK: Algebraic Structures (Specific Groups & Fields), Groups & Fields with Algebraic expressions or functions – L2.** *Mathematical standards for which teaching would be enhanced by teachers' knowledge that operating with specific algebraic expressions or functions requires familiarity with its underlying group/field structure; particularly, where the ability to rigorously justify specific properties of operations within that set aids computational fluency and conceptual understanding*

Geometry & Measurement: Distance Metric

- 2.MD.1 Distance Metric – L2 (Ex: Pyth Thrm & Dist Form as one example; “Dist” from point to line as another; “Dist” on sphere as well (plane travel))
- 6.G.3.- distance formula within polygon - L2.
- 6.NS.7. Absolute value as “distance” – measurement axioms – L2
- 6.NS.8 – constrained distance formula – L2
- 7.G.5. – Angle as a metric. – L2
- 8.G.6-8 – measurement theory, Pythagorean theorem and distance formula. – L2
- F-TF.1. radians representing equivalence between distance metric and angle metric – L3.
- G-C.1-5. Polar coordinate system – L2.
- N-CN.4 – Vector and Polar coordinate systems – L3;
- N-CN.6 – distance metric in complex plane – L2

Black: grades K-4, Blue: grades 5-8, Red: secondary

Calculus of Functions: Rate of Change/Derivative

- 6.RP.2 - Calculus notion of Rate of Change – L1/L2
- 7.RP.2 – rate of change & derivatives in Calculus I – L2
- 8.EE.5 – First derivative, slope, Calculus – L3.
- 8.EE.6. – geometric proof of slope – L3.
- F-IF.6 – Average vs. Instantaneous rate of change – L3.
- F-LE.1-3. Rate of change, derivatives – L3.
- G-C.4. tangent lines in calculus – L3.

Black: grades K-4, Blue: grades 5-8, Red: secondary

Analysis of Number Systems: Base 10 Number System

- K.NBT.1. Place Value in Base 10 – L3.
- K.CC.2. Historical development of 0.
- 1.NBT.1. Historical development of 0.
- 1.NBT.1. Place Value in Base 10 – L3.
- 2.NBT.1-3. Base 10 place value and operations – L3.
- 3.NBT.1 – Historical development of Hindu-Arabic number system, other number systems. L2.
- 4.NBT.1 – Place value, base 10 numbers – L3.
- 4.NF.5 – Historical development of decimal notation – L1;
- 5.NBT.1 – Base 10 – L3;
- 5.NBT.1. Historical development of decimal notation – L2;

Black: grades K-4, Blue: grades 5-8, Red: secondary

Notes

- If a topic is prominent in K-6 but never shows up in HS standards, does that mean secondary teachers don't need it in their mathematical preparation? Or since all earlier grades' standards are taken as prerequisite knowledge for the students, is that also true for teachers?



Implications

- For teaching advanced mathematics content...
- For structure of teacher education programs...
- For pre-service teachers...
- For in-service teachers...
- For mathematics professors...

Notes

- Particularly interested in how to help mathematics professors connect with students who happen to be preservice teachers in general math major classes – what examples or approaches could be incorporated in standard courses that help highlight connections to K-12 content (may be helpful for all students' learning, but especially for responding to future teachers' concerns “why is this class relevant for me??”)

Notes

- How do Levels impact course design? Is anything ever really taught at an L1 in a standard course? Possible support for design of “secondary mathematics from an advanced standpoint” or “mathematics for elementary school teachers” survey courses. Also possible key big ideas to center professional development around for inservice teachers.



Next Steps

- Seeking to measure impact on **planned** teaching practices
- Recent pilot study on impact of learning Group Theory – ongoing data collection into Fall
- Comparison between AMK for CCSS-M & AMK from mathematics teacher preparation textbooks.