THE IMPACT OF GROUP THEORY ON MATHEMATICAL KNOWLEDGE FOR TEACHING

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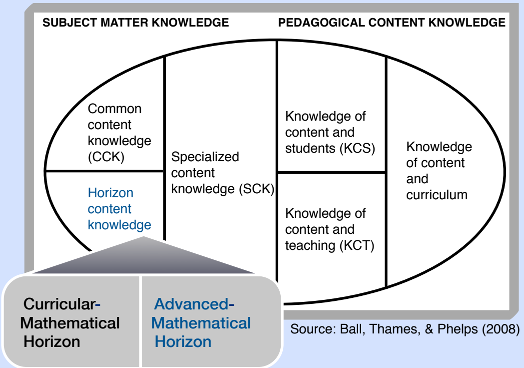
Background

(practice-based approaches to teacher knowledge)

Mathematical Knowledge for Teaching (MKT)



Source: Ball, Thames, & Phelps (2008)

Horizon Content Knowledge (HCK) is an orientation to and familiarity with the discipline (or disciplines) that contribute to the teaching of the school subject at hand, providing teachers with a sense for how the content being taught is situated in and connected to the broader disciplinary territory. HCK includes explicit knowledge of the ways of and tools for knowing in the discipline, the kinds of knowledge and their warrants, and where ideas come from and how "truth" or validity is established… HCK enables teachers to "hear" students, to make judgments about the importance of particular ideas or questions, and to treat the discipline with integrity, all resources for balancing the fundamental task of connecting learners to a vast and highly developed field.
(Jakobsen, Thames, Ribeiro, & Delaney, 2012)

Knowledge for Algebra Teaching (KAT)

CATEGORIES OF TEACHER KNOWLEDGE
- Knowledge of School Algebra
- **Knowledge of Advanced Mathematics**
- Mathematics-for-Teaching Knowledge

USE OF KNOWLEDGE IN TEACHING
- **Trimming:** removing complexity while maintaining integrity
- **Decompressing:** Unpacking complexity in ways that make it comprehensible
- **Bridging:** Making connections across topics, assignments, representations, and domains

Source: McRory, Floden, Ferrini-Mundy, Reckase, & Senk (2012)

Research Framework

Instructional Approach used in Study

Looking at K-12 mathematics from an advanced perspective. Connecting algebraic structures to early algebra learning.

Teacher Demographics

N=6 teachers (2 ES, 2 MS, 2 HS) Wide range of teaching experience and mathematics backgrounds

(Advanced) Horizon Content Knowledge

Knowledge of Group Theory

A set of elements (S), under a binary operation (*), forms a group (S,*) if:
1. Closure: $\forall a,b \in S, a*b \in S$
2. Associative: $\forall a,b,c \in S, (a*b)*c = a*(b*c)$
3. Identity: $\exists e \in S, s.t., \forall a \in S, e*a = a*e = a$
4. Inverse: $\forall a \in S, \exists a^{-1} \in S, s.t., a*a^{-1} = a^{-1}*a = e$

Pre **Content Assessment:** Post
Arithmetic Properties
Abstract Algebra
Mathematical Explanations
Concept Maps

Pre **Planned Practice** Post
Lesson Plans
Interviews
Beliefs Survey

Teaching of K-12 Mathematics

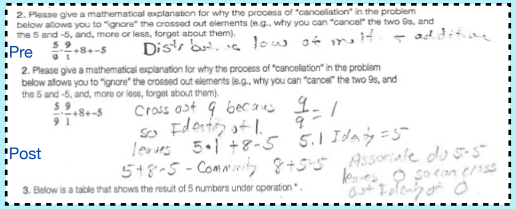
CCSSM Standards such as:

1.OA.3. Apply properties of operations as strategies to add and subtract.[2] *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)*
4.NF.1. Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models... Use this principle to recognize and generate equivalent fractions.
6.EE.7. Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.
A-APR.1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
F-BF.4.A. (+) Verify by composition that one function is the inverse of another.

Pre **Reflective Practice** Post
Interviews
Beliefs Surveys

Pre **Enacted Practice** Post
Videotaped Lessons

Selected Results

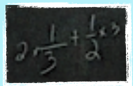


Impact on Practice: Decompressing

- Appreciation for value of the associative and commutative properties in mental arithmetic
- Development of tasks that help students discover important arithmetic properties (e.g. pattern building)
- Explicit documentation of the inverse and identity roles in "cancellation" for solving equations
- Increased desire/intention to require students to justify properties of operations
- Increased desire/intention to explain mathematical reasoning behind each step in an equation solving process

Impact on Practice: Trimming & Bridging

PRE
- Inaccurate trimming; reliance on rules & quick-tricks without meaning.
- "what you do to the top you do to the bottom [getting common denominators for adding fractions]"



POST
- Recognition of importance of being careful with presentation, symbols & terminology when trimming
- Increased use of mathematically precise vocabulary, with the goals of unpacking meaning and of setting students up for future studies, e.g., bridging

Pedagogical Changes

- "I think one of the things from the summer that I really got was taking a step back and **letting the students think about numbers and think about what is going on** as opposed to me jumping in and telling them… like you did in our class, you gave us time to struggle with what is going on."

Attitudinal Changes

- "….because you know what's coming, where you're going with it, but **you understand it better, and you feel more comfortable teaching it.**"
- "I think I have worked better at trying to explain why…**you made me understand *why* better so I've been able to do a better job with that.**"

Remarks

- Further exploration across multiple instructional approaches used for delivery of advanced content is warranted
- Analysis is still ongoing of the relative impact of participants' increased content mastery and attitudinal changes on teaching practices
- Consider use of concept maps as a tool for both further content assessment and prompting reflection on practice (e.g., participant interviews)