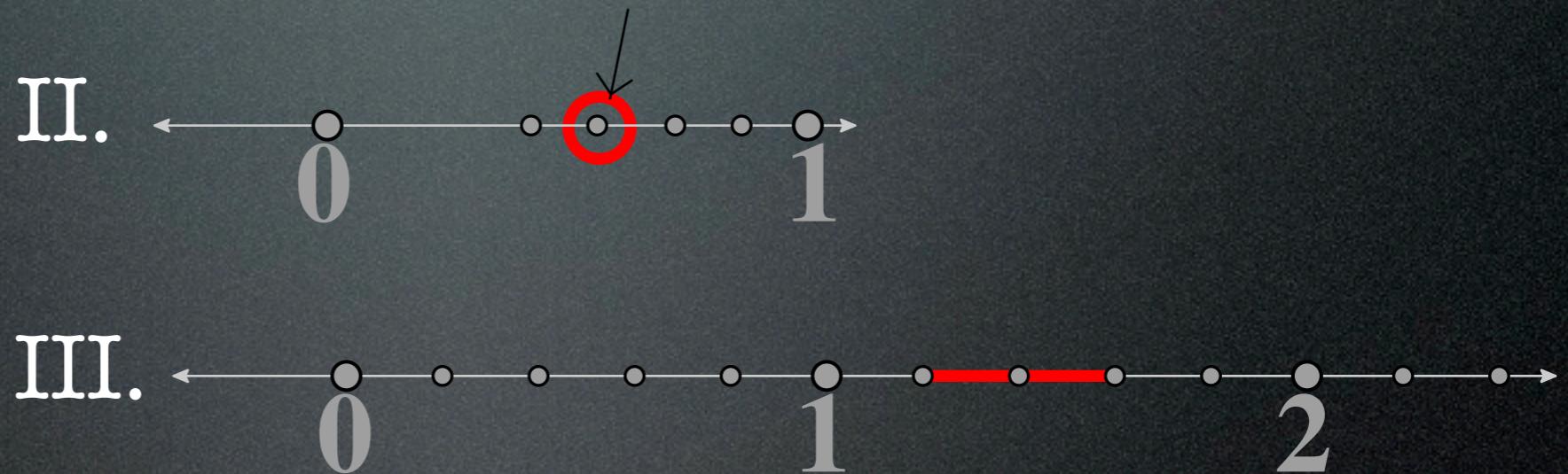
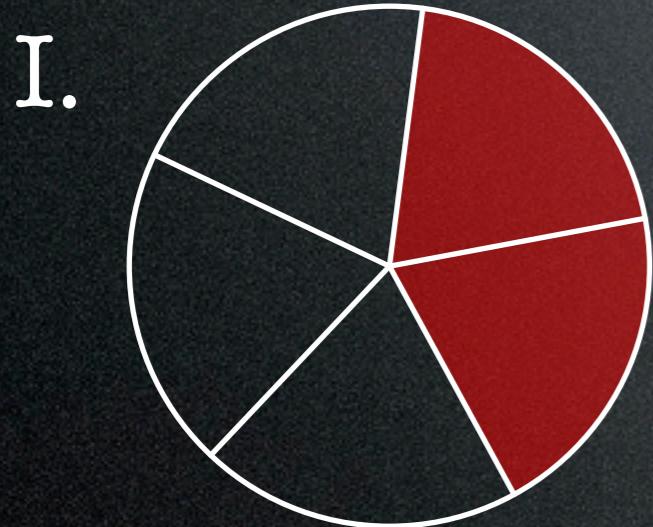


Systems-level Content Development: Establishing Learning Progressions

RME Research to Practice Conference
Nick Wasserman, Janie Schielack
24 February 2012

A Math Question

Which of the following are correct representations of $2/5$?



A. I, III only

B. I only

C. II only

D. I, II, III

What are Learning Progressions?

What are Learning Progressions?

LEARNING TRAJECTORY DISPLAY COMMON CORE STATE STANDARDS FOR MATHEMATICS, GRADES 6-8

THE PRACTICES OF MATHEMATICS: 1 Make sense of problems and persevere in solving them. 2 Reason abstractly and quantitatively. 3 Construct viable arguments and critique the reasoning of others. 4 Model with mathematics. 5 Use appropriate tools strategically. 6 Attend to precision. 7 Look for and make use of structure. 8 Look for and express regularity in repeated reasoning.

Ratio and Proportional Relationships and Percent

CONTENT STRAND	GRADE 6	GRADE 7	GRADE 8	CONTENT STRAND
Ratio, Rate, and Slope	<p>6.RP.1 Understand the concept of ratio and unit rate to describe a ratio relationship between two quantities. For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote Castille received, Crandall received three votes.”</p> <p>6.RP.2 Understand the concept of a unit rate $\frac{a}{b}$ associated with a ratio $a:b$, where $b \neq 0$; compute unit rates in context of a ratio relationship.</p> <p>6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. For example, find a missing value in a ratio table given two ratios in the table.</p> <p>6.RP.4 Find a percent of a quantity as a rate per 100; solve problems involving finding the whole, given a part and the percent.</p> <p>6.RP.5 Use ratio and rate reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.</p>	<p>7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, a person who walks 1/2 mile in each 1/4 hour, compute the unit rate as walking 3 miles per hour, equivalently 3/2 miles per 1/2 hour.</p> <p>7.RP.2 Recognize and represent proportional relationships between quantities; decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in tables, graphs, or equations; distinguish proportional relationships from other relationships that do not have constant ratios.</p> <p>7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. For example, if it takes 24 minutes to mix a batch of cookie dough and 30 minutes to bake a batch of cookies, how many batches of cookies can be baked after 150 minutes?</p>	<p>8.EE.2 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.</p> <p>8.EE.5 Graph linear equations. For example, graph the equation $y = 3x + 5$ as shown below, showing the origin and some other points on the line.</p>	Ratio, Rate, and Slope
Word Problems	<p>6.RP.1 Understand the concept of ratio and unit rate to describe a ratio relationship between two quantities. For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote Castille received, Crandall received three votes.”</p> <p>6.RP.2 Understand the concept of a unit rate $\frac{a}{b}$ associated with a ratio $a:b$, where $b \neq 0$; compute unit rates in context of a ratio relationship.</p> <p>6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. For example, find a missing value in a ratio table given two ratios in the table.</p> <p>6.RP.4 Find a percent of a quantity as a rate per 100; solve problems involving finding the whole, given a part and the percent.</p> <p>6.RP.5 Use ratio and rate reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.</p>	<p>7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, a person who walks 1/2 mile in each 1/4 hour, compute the unit rate as walking 3 miles per hour, equivalently 3/2 miles per 1/2 hour.</p> <p>7.RP.2 Recognize and represent proportional relationships between quantities; decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in tables, graphs, or equations; distinguish proportional relationships from other relationships that do not have constant ratios.</p> <p>7.RP.3 Use proportional relationships to solve multistep ratio and percent problems. For example, if it takes 24 minutes to mix a batch of cookie dough and 30 minutes to bake a batch of cookies, how many batches of cookies can be baked after 150 minutes?</p>	<p>8.EE.2 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.</p> <p>8.EE.5 Graph linear equations. For example, graph the equation $y = 3x + 5$ as shown below, showing the origin and some other points on the line.</p>	Word Problems

Rational Number System and Operations and Introduction to Irrationals

Content Strand	Grade 6	Grade 7	Grade 8	Content Strand
Whole Numbers, Rational Numbers, and Irrational Numbers	<p>GRADE 6</p> <p>6.NS.3. Fluently divide multi-digit numbers using the standard algorithm.</p> <p>6.NS.4. Find the greatest common factor of two whole numbers less than or equal to 120 and the least common multiple of two whole numbers less than or equal to 120. Use the distributive property to express a sum of two whole numbers less than or equal to 120 as a multiple of a sum of one-digit numbers (e.g., $36 + 9 = 9(4 + 1)$).</p> <p>For example, express 28×6 as $4 \times (7 \times 6)$.</p>	<p>GRADE 7</p> <p>7.NS.2.a. Convert a rational number to a decimal using long division; know that a decimal form of a rational number terminates in 0 or eventually repeats.</p>	<p>GRADE 8</p> <p>8.NS.1. Know that numbers that are not rational are called irrational. Understand informally that every real number is either rational or irrational; that irrational numbers are approximated by rational numbers.</p> <p>8.NS.2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2).</p> <p>For example, by truncating the decimal expansion of π, determine that π is between 3 and 4, between 14 and 15, and explain how to continue on to get better approximations.</p>	Whole Numbers, Rational Numbers, and Irrational Numbers
Negative Numbers	<p>6.NS.5. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative numbers) in real-world contexts. Explain meanings of positive and negative numbers in real-world contexts, explaining the meaning of 0 in each situation.</p> <p>6.NS.6. A -1 is the additive inverse of 1; for example, $-1 + 1 = 0$. Show that the distance between two integers on the number line is the absolute value of their difference, specifically, that if a and b are integers, then $a - b$ is the distance between a and b on the number line. Recognize that the signs of the digits of a number in the number representation do not affect the value of the number.</p> <p>6.NS.7. Understand ordering and absolute value of rational numbers in real-world contexts. For example, interpret negative integer temperatures as locations below freezing in a coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across the origin, or are on a line through the origin, depending on the context in which the numbers are used.</p> <p>6.NS.8. Understand a rational number as a point on the number line.</p>	<p>7.NS.1.b. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. For example, show that $-1 + 2 = 1$ by writing $-1 + 2 = 0 + 2 = 2$.</p> <p>7.NS.1.c. Apply properties of operations as strategies to add and subtract rational numbers.</p> <p>7.NS.2.a. Apply and extend previous understandings of multiplication and division and fractions to multiply and divide rational numbers by fractions. Show that $(ab)c = a(bc)$ for the case of rational numbers by repeating arguments similar to those given for the case of integers and ratios. Interpret products of rational numbers by describing real-world contexts.</p> <p>7.NS.2.b. Apply properties of operations as strategies to multiply rational numbers.</p>	<p>8.NS.2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2).</p> <p>For example, by truncating the decimal expansion of π, determine that π is between 3 and 4, between 14 and 15, and explain how to continue on to get better approximations.</p>	Negative Numbers
Locating and Operating with Rational Numbers in 1D and 2D Space	<p>6.NS.1. Distinguish between the place value of a digit in one position and another in another position in a multi-digit number using expanded form.</p> <p>6.NS.2. Understand that integer opposites are a pair of integers that have the same absolute value but different signs. For example, -3 and 3 are integer opposites, and 15 and -15 are integer opposites.</p> <p>6.NS.3. Fluently divide multi-digit numbers using the standard algorithm.</p> <p>6.NS.4. Find the greatest common factor of two whole numbers less than or equal to 120 and the least common multiple of two whole numbers less than or equal to 120. Use the distributive property to express a sum of two whole numbers less than or equal to 120 as a multiple of a sum of one-digit numbers (e.g., $36 + 9 = 9(4 + 1)$).</p> <p>For example, express 28×6 as $4 \times (7 \times 6)$.</p> <p>6.NS.5. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative numbers) in real-world contexts. Explain meanings of positive and negative numbers in real-world contexts, explaining the meaning of 0 in each situation.</p> <p>6.NS.6. A -1 is the additive inverse of 1; for example, $-1 + 1 = 0$. Show that the distance between two integers on the number line is the absolute value of their difference, specifically, that if a and b are integers, then $a - b$ is the distance between a and b on the number line. Recognize that the signs of the digits of a number in the number representation do not affect the value of the number.</p> <p>6.NS.7. Understand ordering and absolute value of rational numbers in real-world contexts. For example, interpret negative integer temperatures as locations below freezing in a coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across the origin, or are on a line through the origin, depending on the context in which the numbers are used.</p> <p>6.NS.8. Understand a rational number as a point on the number line.</p>	<p>7.NS.2.a. Apply and extend previous understandings of multiplication and division and fractions to multiply and divide rational numbers by fractions. Show that $(ab)c = a(bc)$ for the case of rational numbers by repeating arguments similar to those given for the case of integers and ratios. Interpret products of rational numbers by describing real-world contexts.</p> <p>7.NS.2.b. Apply properties of operations as strategies to multiply rational numbers.</p> <p>7.NS.3. Solve real-world and mathematical problems involving the four operations with rational numbers (including mixed numbers) and with negative rational numbers in real-world contexts.</p>	<p>8.NS.2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2).</p> <p>For example, by truncating the decimal expansion of π, determine that π is between 3 and 4, between 14 and 15, and explain how to continue on to get better approximations.</p>	Locating and Operating with Rational Numbers in 1D and 2D Space
Word Problems and Rational Numbers	<p>6.NS.2. Understand absolute value of rational numbers, a, as the distance of a from zero on the number line; interpret negative rational numbers as locations less than zero.</p> <p>6.NS.3. Fluently divide multi-digit numbers using the standard algorithm.</p> <p>6.NS.4. Find the greatest common factor of two whole numbers less than or equal to 120 and the least common multiple of two whole numbers less than or equal to 120. Use the distributive property to express a sum of two whole numbers less than or equal to 120 as a multiple of a sum of one-digit numbers (e.g., $36 + 9 = 9(4 + 1)$).</p> <p>For example, express 28×6 as $4 \times (7 \times 6)$.</p> <p>6.NS.5. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative numbers) in real-world contexts. Explain meanings of positive and negative numbers in real-world contexts, explaining the meaning of 0 in each situation.</p> <p>6.NS.6. A -1 is the additive inverse of 1; for example, $-1 + 1 = 0$. Show that the distance between two integers on the number line is the absolute value of their difference, specifically, that if a and b are integers, then $a - b$ is the distance between a and b on the number line. Recognize that the signs of the digits of a number in the number representation do not affect the value of the number.</p> <p>6.NS.7. Understand ordering and absolute value of rational numbers in real-world contexts. For example, interpret negative integer temperatures as locations below freezing in a coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across the origin, or are on a line through the origin, depending on the context in which the numbers are used.</p> <p>6.NS.8. Understand a rational number as a point on the number line.</p>	<p>7.NS.2.c. Solve real-world and mathematical problems involving the four operations with rational numbers (including mixed numbers).</p> <p>7.NS.3. Solve real-world and mathematical problems involving the four operations with rational numbers (including mixed numbers) and with negative rational numbers in real-world contexts.</p>	<p>8.NS.2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2).</p> <p>For example, if a person making \$20 an hour gets a 10% raise, she will make an additional $1/10$ of her salary or $\\$2$ more each hour. If you earn $\\$15$ an hour and get a 10% raise, how much do you earn per hour now? If you're buying a $1/2$-inch nail but get a $3/8$-inch nail, how much longer is the nail? If you buy a $1/2$-pound bag of dog food at $\\$2.50$ a pound and it requires $1/2$ cup of food per 10 pounds of dog food, how much dog food will be left over if the dog eats $1/4$ cup a day?</p>	Word Problems in Any of the Four Operations

Algebraic Reasoning

Content Strand	Grade 6	Grade 7	Grade 8	Content Strand
Exponents, Roots, and Scientific Notation	<p>6.EE.1 Write and evaluate numerical expressions involving whole-number exponents.</p>	<p>6.EE.1 Know and apply the properties of integers to generate equivalent integer expressions. For example, $25 \cdot (-3) = -3 \cdot 25 = -75$.</p>	<p>6.EE.2 Write numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much the one is than the other. For example, estimate the population of the United States to be 3×10^9 and the population of the world to be more than 7 times larger.</p>	Exponents, Roots, and Scientific Notation
Expressions	<p>6.EE.2a Write, read, and evaluate expressions in which letters stand for numbers. For example, express the calculation “Subtract t from s” as $s - t$; evaluate the expression $5 + 3(1 - x)$ when $x = 0.4$. Write expressions that record operations with numbers and letters standing for numbers. For example, express the calculation “Subtract t from s” as $s - t$.</p> <p>6.EE.2b Evaluate expressions using properties of operations. Include expressions combined by addition or subtraction of unlike terms; include expressions containing products of two or more variables, where coefficients are rational numbers; include expressions containing parentheses; and include rational exponents provided that their bases are nonnegative real numbers.</p> <p>6.EE.2c Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression $3(2 + x)$ to produce the equivalent expression $6 + 3x$; apply properties of operations to $y + y + y$ to produce the equivalent expression $3y$.</p> <p>6.EE.2d Identify when two expressions are equivalent (i.e., when the two expressions name the same number), based on the properties of operations and the equality relationship between two expressions. For example, identify $3(x + 2y)$ and $2(3x + 6y)$ as equivalent because they name the same number regardless of which numbers x and y are substituted into them.</p> <p>6.EE.2e Use variables to represent numbers and write expressions when solving a real-world or mathematical problem, understanding that a variable can represent an unknown number, or depending on the context, a changing quantity.</p> <p>6.EE.2f Understand solving an equation or inequality as a process of answering a question which values make the equation or inequality true. Evaluate the reasonableness of the solution in a real-world context.</p> <p>6.EE.2g Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for p, q and x, where p, q are given numbers, and x is a variable.</p> <p>6.EE.2h Write an inequality of the form $x > a$ or $x < a$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > a$ or $x < a$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.</p>	<p>7.EE.1 Apply properties of operations to add, subtract, factor, and expand linear expressions with rational coefficients.</p> <p>7.EE.2 Understand that rewriting an expression in different forms is a problem-solving strategy that can lead to equivalent expressions whose value is easier or more familiar to work with. For example, $x + 0.4x = 1.4x$ is equivalent to $7(29 + 1) = 7 \cdot 30$.</p> <p>7.EE.3 Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which parts of the arguments justify which conclusions. For example, the perimeter of a rectangle is 56 cm. Its length is 6 cm. What is its width?</p> <p>7.EE.4a Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form $p + q = c$ and $p = q - c$, where p, q, and c are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution. For example, as a customer, you are paid \$0.02 per plus \$0.03 per call. This week you want to pay at least \$20.00. Write an inequality for the number of calls you need to make, and describe the solution.</p>	<p>8.EE.1 Know and apply the properties of integers to generate equivalent integer expressions. For example, $25 \cdot (-3) = -3 \cdot 25 = -75$.</p> <p>8.EE.2 Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much the one is than the other. For example, estimate the population of the United States to be 3×10^9 and the population of the world to be more than 7 times larger.</p> <p>8.EE.3 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used, using scientific notation and choosing units of appropriate size for measurements of very large or very small quantities. For example, interpret the meaning of 7.2×10^{-3} meters as 7.2 millimeters per year for nuclear spreading. Interpret scientific notation that has been generated by technology.</p> <p>8.EE.4 Use square root and cube root symbols to represent solutions to equations of the form $x^p = a$ and $x^q = a$, where p and q are rational numbers and a is a positive real number. Evaluate square roots of small perfect squares. Note that \sqrt{a} is irrational.</p>	Expressions
One Variable Equations, Inequalities, and Word Problems		<p>8.EE.7 Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which parts of the arguments justify which conclusions. For example, $x + 0.4x = 1.4x$ is equivalent to $7(29 + 1) = 7 \cdot 30$.</p> <p>b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting the terms.</p>		One Variable Equations, Inequalities, and Word Problems
Simultaneous Linear Equations		<p>8.EE.8a Analyze and solve pairs of simultaneous linear equations. a. Determine that the solution to a system of two linear equations in two variables corresponds to the point(s) of intersection of their graphs, because points of intersection are the solutions to the systems of equations they define; solve pairs of linear equations in two variables algebraically, and estimate solutions by graphing the equations.</p> <p>8.EE.8b Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.</p> <p>8.EE.8c Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is a line) or by calculating data points. Sketch a graph of a nonlinear function that exhibits a repeating cycle over an interval and determine whether the function is periodic.</p> <p>8.EE.8d Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).</p> <p>8.EE.8e Given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.</p>		Introduction to Functions and Linear Functions
Introduction to Functions and Linear Functions	<p>6.EE.9 Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other. For example, in a problem involving motion at constant speed, list the known constants and independent variables using a right triangle, and write the equation to represent the relationship between distance and time.</p> <p>For example, in a problem involving motion at constant speed, list the known constants and independent variables using a right triangle, and write the equation to represent the relationship between distance and time.</p>	<p>8.EE.8a Analyze and solve pairs of simultaneous linear equations. a. Determine that the solution to a system of two linear equations in two variables corresponds to the point(s) of intersection of their graphs, because points of intersection are the solutions to the systems of equations they define; solve pairs of linear equations in two variables algebraically, and estimate solutions by graphing the equations.</p> <p>8.EE.8b Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.</p> <p>8.EE.8c Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is a line) or by calculating data points. Sketch a graph of a nonlinear function that exhibits a repeating cycle over an interval and determine whether the function is periodic.</p> <p>8.EE.8d Given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.</p> <p>8.EE.8e Given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.</p>		Introduction to Functions and Linear Functions

Geometry

What are Learning Progressions?

4

Counting and Cardinality

Several progressions originate in knowing number names and the count sequence.^{K.CC.1}

From saying the counting words to counting out objects Students usually know or can learn to say the counting words up to a given number before they can use these numbers to count objects or to tell the number of objects. Students become fluent in saying the count sequence so that they have enough attention to focus on the pairings involved in counting objects. To count a group of objects, they pair each word said with one object.^{K.CC.4a} This is usually facilitated by an indicating act (such as pointing to objects or moving them) that keeps each word said in time paired to one and only one object located in space. Counting objects arranged in a line is easiest; with more practice, students learn to count objects in more difficult arrangements, such as rectangular arrays (they need to ensure they reach every row or column and do not repeat rows or columns); circles (they need to stop just before the object they started with); and scattered configurations (they need to make a single path through all of the objects).^{K.CC.5} Later, students can count out a given number of objects,^{K.CC.5} which is more difficult than just counting that many objects, because counting must be fluent enough for the student to have enough attention to remember the number of objects that is being counted out.

From subitizing to single-digit arithmetic fluency Students come to quickly recognize the cardinalities of small groups without having to count the objects; this is called perceptual subitizing. Perceptual subitizing develops into conceptual subitizing—recognizing that a collection of objects is composed of two subcollections and quickly combining their cardinalities to find the cardinality of the collection (e.g., seeing a set as two subsets of cardinality 2 and saying ‘four’). Use of conceptual subitizing in adding and subtracting small numbers progresses to supporting steps of more advanced methods for adding, subtracting, multiplying, and dividing single-digit numbers (in several OA standards from Grade 1 to 3 that culminate in single-digit fluency).

From counting to counting on Students understand that the last number name said in counting tells the number of objects counted.^{K.CC.4b} Prior to reaching this understanding, a student who is asked ‘How many kittens?’ may regard the counting performance itself as the answer, instead of answering with the cardinality of the set. Experience with counting allows students to discuss and come to understand the second part of K.CC.4b—that the number of objects is the same regardless of their arrangement or the order in which they were counted. This connection will continue in Grade 1 with the

K.CC.1 Count to 100 by ones and by tens.

K.CC.4a Understand the relationship between numbers and quantities; connect counting to cardinality.

- a When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.

K.CC.5 Count to answer “how many?” questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

K.CC.4b Understand the relationship between numbers and quantities; connect counting to cardinality.

- b Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.

What are Learning Progressions?

Developmental Levels for Recognizing Number and Subitizing (Instantly Recognizing)

Age Range	Level Name	Level	Description
2	Small Collection Namer	1	Names groups of one to two, sometimes three. For example, shown a pair of shoes, child says "Two shoes."
3	Maker of Small Collections	2	Nonverbally makes a small collection (no more than 4, usually 1-3) with the same number another collection. For example, when shown a collection of 3, makes another collection of 3.
4	Perceptual Subitizer to 4	3	Instantly recognizes collections up to 4 when briefly shown and verbally names the number of items. For example, when shown 4 objects briefly, says "four."
5	Perceptual Subitizer to 5	4	Instantly recognize briefly shown collections up to 5 and verbally name the number of items. For example, when shown 5 objects briefly, says "5."
5	Conceptual Subitizer to 5+	5	Verbally labels all arrangements to about 5, when shown only briefly. For example, says "Five! Why? Because I saw three and two and so I said five."

Age Range	Level Name	Level	Description
5	Conceptual Subitizer to 10	6	Verbally label most briefly shown arrangements to 6, then up to 10, using groups. For example, says, "In my mind, I made two groups of 3 and one more, so 7."
6	Conceptual Subitizer to 20	7	Verbally label structured arrangements up to 20, shown only briefly, using groups. For example, says, "I saw three fives, so 5, 10, 15."
7	Conceptual Subitizer with Place Value and Skip Counting	8	Verbally label structured arrangements shown only briefly, using groups, skip counting, and place value. For example, says, "I saw groups of ten and twos, so 10, 20, 30, 40, 42, 44, 46...46!"
8+	Conceptual Subitizer with Place Value and Multiplication	9	Verbally label structured arrangements shown only briefly, using groups, multiplication, and place value. For example, says, "I saw groups of ten and threes so I thought, five tens is 50 and four threes is 12, so 62 in all."

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4. Learning Performances at each Level that articulate students' performance capability
5. Assessments that measure student development along the progression

A Science example

Solar System Progression: from Wilson (2009)

Level	Description
5 8 th grade	<p>Student is able to put the motions of the Earth and Moon into a complete description of motion in the Solar System which explains:</p> <ul style="list-style-type: none">• the day/night cycle• the phases of the Moon (including the illumination of the Moon by the Sun)• the seasons
4 5 th grade	<p>Student is able to coordinate apparent and actual motion of objects in the sky. Student knows that</p> <ul style="list-style-type: none">• the Earth is both orbiting the Sun and rotating on its axis• the Earth orbits the Sun once per year• the Earth rotates on its axis once per day, causing the day/night cycle and the appearance that the Sun moves across the sky• the Moon orbits the Earth once every 28 days, producing the phases of the Moon <p>COMMON ERROR: Seasons are caused by the changing distance between the Earth and Sun.</p> <p>COMMON ERROR: The phases of the Moon are caused by a shadow of the planets, the Sun, or the Earth falling on the Moon.</p>
3	<p>Student knows that:</p> <ul style="list-style-type: none">• the Earth orbits the Sun• the Moon orbits the Earth• the Earth rotates on its axis <p>However, student has not put this knowledge together with an understanding of apparent motion to form explanations and may not recognize that the Earth is both rotating and orbiting simultaneously.</p> <p>COMMON ERROR: It gets dark at night because the Earth goes around the Sun once a day.</p>
2	<p>Student recognizes that:</p> <ul style="list-style-type: none">• the Sun appears to move across the sky every day• the observable shape of the Moon changes every 28 days <p>Student may believe that the Sun moves around the Earth.</p> <p>COMMON ERROR: All motion in the sky is due to the Earth spinning on its axis.</p> <p>COMMON ERROR: The Sun travels around the Earth.</p> <p>COMMON ERROR: It gets dark at night because the Sun goes around the Earth once a day.</p> <p>COMMON ERROR: The Earth is the center of the universe.</p>
1	<p>Student does not recognize the systematic nature of the appearance of objects in the sky. Students may not recognize that the Earth is spherical.</p> <p>COMMON ERROR: It gets dark at night because something (e.g., clouds, the atmosphere, "darkness") covers the Sun.</p> <p>COMMON ERROR: The phases of the Moon are caused by clouds covering the Moon.</p> <p>COMMON ERROR: The Sun goes below the Earth at night.</p>
0	No evidence or off-track

A Math example

Equipartitioning:
Important for rational number & fraction development

Case	Equipartitioning Progress Variable
D	1.8 m objects shared among p people, $m > p$
C	1.7 m objects shared among p people, $p > m$
B	1.6 Splitting a continuous whole object into odd # of parts ($n > 3$)
B	1.5 Splitting a continuous whole object among $2n$ people, $n > 2$, & $2n \neq 2^i$
B	1.4 Splitting continuous whole objects into three parts
B	1.3 Splitting continuous whole objects into 2^n shares, with $n > 1$
A	1.2 Dealing discrete items among $p = 3 - 5$ people, with no remainder; mn objects, $n = 3, 4, 5$
A, B	1.1 Partitioning using 2-split (continuous and discrete quantities)

from Mojica & Confrey (2009)

MStar Goal

- Create a Diagnostic Assessment for struggling learners
- Develop and Use Learning Progressions as the framework for Diagnostic
- Better understand “why” students struggle, not “what” they struggle with
- Some of the issues

Your Turn

Learning Goal:

For students to be able to represent a variety of number patterns with tables, graphs, words, and symbolic rules

Your Turn

BELOW PROFICIENCY			PROFICIENT	ADVANCED
Less Complex			More Complex	
The student will:				
<p>• Determine the next 3 values in a given sequence of numbers (e.g., given the sequence “3, 7, 11, 15 ...” conclude that the next three values will be 19, 23, and 27).</p> <p>-----</p>	<p>• Organize the values in a given sequence using a table and/or graph (e.g., where “x-value” represents the placement in the sequence (i.e., 1 for the 1st term, 2 for the 2nd term, etc.) and the y-value represents the value of the term). [NOTE: Include different kinds of patterns, such as numerical, spatial, and recursive.]</p> <p>-----</p>	<p>• Organize the values in a given sequence using a table and/or graph and determine the recursive pattern in the sequence (e.g., given the sequence “3, 7, 11, 15 ...” conclude that the next number is obtained by adding 4 to the previous value)</p> <p>-----</p>	<p>• Organize the values in a given sequence using a table and/or graph and be able to state an explicit rule to find the value of the nth term either symbolically or verbally (e.g., given the sequence “3, 7, 11, 15 ...” conclude that the rule is $y=4x-1$, or an equivalent form, or verbally describing that you have to multiply the term number by 4 and then subtract 1).</p> <p>-----</p>	<p>• Explain how a table of values can be used to determine whether a function is linear or nonlinear. Explanation should include an example to demonstrate each.</p> <p>-----</p>

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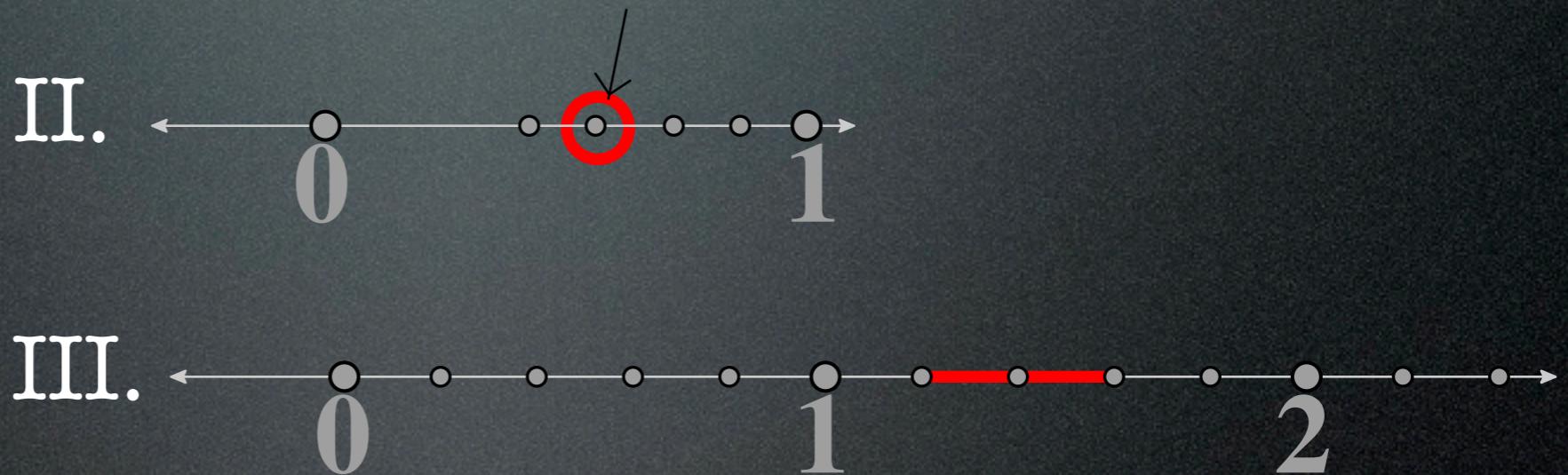
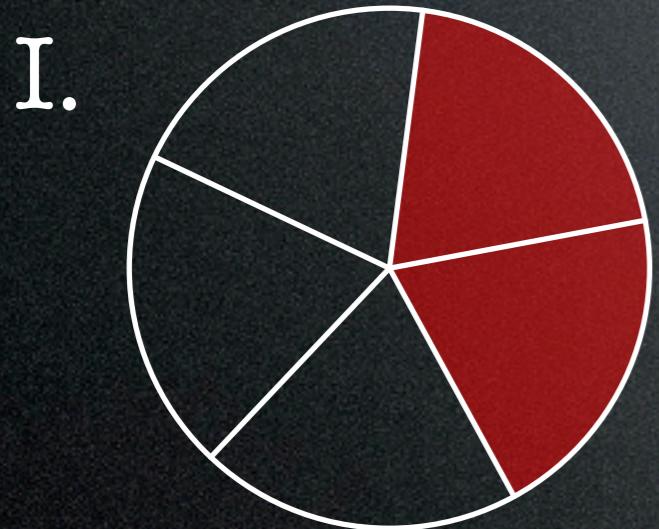
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How do LPs help?

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Which of the following are correct representations of $2/5$?



- A. I, III only
- B. I only
- C. II only
- D. I, II, III

How do LPs help?

1.2

- ii. The student understands that a number has a specific location on the number line based on what is "next" in the list of numbers (ordinal), and that numbers represent a distance or quantity from 0 (cardinal). **(M)**
Understands the end point as the distance, regardless of the beginning point
- iv. The student understands the magnitude of "common" fractions (e.g. $1/2$, $1/4$), and use "common" fractions to estimate magnitude or distance.

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2.2

- i. The student will be able to partition **shapes** into equal regions (with equal areas) using paper strips and pictorial representations. The student recognizes that shapes of different sizes can be partitioned equally and still represent unit fractions. **(M)** Does not recognize that for fraction models involving area, two parts may look different but hold the same relationship to the whole
- ii. The student makes the connection that a whole is composed of 2 halves, 3 thirds, 4 fourths, and that the number of parts is the denominator of the unit fraction. The student can label each part of the partitioned whole as a fraction.

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1.3	<p>iv.a. The student understands the magnitude or distance between two numbers is related to counting the number of equal intervals. (M) Views fractions only as part:whole relationships and not as numbers in their own right (e.g. they view $\frac{1}{4}$ in relation to 1, but not as its own number, $\frac{1}{4}$). (E) Incorrectly "counts" intervals between $\frac{2}{5}$ and $\frac{6}{5}$ as "4."</p> <p>iv.b. The student understands that fractions, $\frac{1}{b}$, are located by dividing 1 into b equal intervals (e.g. $\frac{1}{4}$ as dividing 1 into 4 equal intervals). The student will be able to make the connection that if the numerator is larger than the denominator then that improper fraction is greater than 1, and if the numerator is smaller than the denominator then that fraction is less than 1. (i.e. $\frac{3}{3}=1$, so $\frac{5}{3}>1$ and $\frac{3}{5}<1$). (M) Does not grasp that fractions are a quantity (cardinal), measured as a distance from 0.</p>

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Theoretical Distribution

Grade 5 mathematics

4.

$$47 \overline{)1325}$$

- a. 28 R 9
- b. 28 R 1
- c. 30 R 15
- d. 28 R 19

Theoretical Distribution

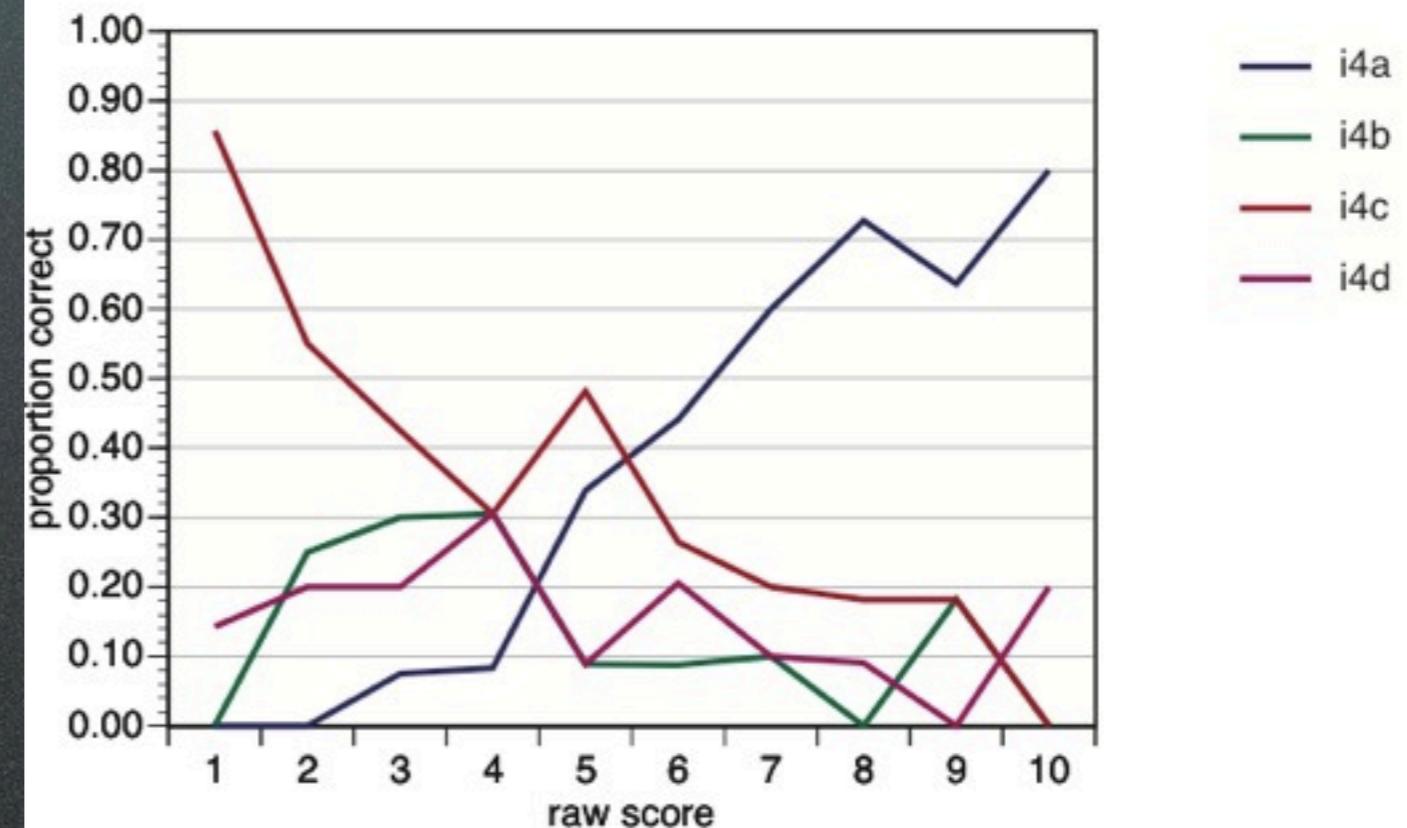
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observed



MStar Process

A Learning Progression, according to Corcoran, Mosher, and Rogat (2009), contains:

MStar Process

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1. Target Learning Goals

MStar Process

A Learning Progression, according to Corcoran, Mosher, and Rogat (2009), contains:

1. Target Learning Goals
2. Reportable Outcomes, key concepts

MStar Process

A Learning Progression, according to Corcoran, Mosher, and Rogat (2009), contains:

1. Target Learning Goals
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3. Progress Variables that are developed over time
4. Intermediate Levels of Achievement that progress toward mastery
5. Learning Performances at each Level that articulate students' performance capability
6. Assessments that measure student development along the progression

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MStar Progressions

LP1: Understanding Positive Rational Numbers, their Representations, and their Uses

LP2: Understanding Variable Expressions, and their Applications

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LP1

Understanding Positive Rational Numbers, their Representations, and their Uses

Magnitude

Equipartitioning

Decomposition

LP1

Understanding Positive Rational Numbers, their Representations, and their Uses

Magnitude

Equivalent
Fractions

Equipartitioning

Decimals

Decomposition

Comparing
Fractions

Conversion
between
Representations

LP1

Understanding Positive Rational Numbers, their Representations, and their Uses

Magnitude

Equivalent
Fractions

Meaning of
Addition

Equipartitioning

Decimals

Meaning of
Multiplication

Decomposition

Comparing
Fractions

Meaning of
Division

Conversion
between
Representations

Proportional
Reasoning

LP2

Understanding Variable Expressions, and their Applications

Variables as
Unknown Quantity

Evaluate

Verbal Translations
of Expressions and
Equations

Simplifying
Expressions

LP2

Understanding Variable Expressions, and their Applications

Variables as
Unknown Quantity

Evaluate

Verbal Translations
of Expressions and
Equations

Simplifying
Expressions

Relationships
between
Expressions

Solving Equations

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Example of Sublevels

Equivalent Fractions Progression

Level Description	Misconceptions
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Example of Sublevels

Equivalent Fractions Progression

	Level Description	Misconceptions
4.1	i. Given a diagram, the student understands that different fractions can represent the same magnitude.	i. Is not able to generate equivalent fractions without being given a diagram.

Example of Sublevels

Equivalent Fractions Progression

	Level Description	Misconceptions
4.1	i. Given a diagram, the student understands that different fractions can represent the same magnitude.	i. Is not able to generate equivalent fractions without being given a diagram.
4.2	i. Given a diagram, the student can recognize a model that represents an equivalent fraction. The student understands that equivalent fractions will always occupy the same point on the number line. ii. The student understands that the number and size of the parts differ even though the two fractions themselves are equivalent. (e.g. $\frac{3}{4}$ has 3 "bigger" parts, and $\frac{6}{8}$ has 6 "smaller" parts.)	i. Cannot generate equivalent fractions, can only recognize equivalence when given the models. When asked if two fractions are equivalent, they make mistakes based on estimating partitions (e.g. conclude that $\frac{3}{5}$ and $\frac{6}{10}$ are not equivalent because in their drawing the points did not exactly match up) ii. Does not recognize when "denominators" are easily related as multiples of each other. (e.g. that denominators of 6 and 12 are easily related; but 3 and 5 are not as easily related.)

Example of Sublevels

Equivalent Fractions Progression

	Level Description	Misconceptions
4.1	i. Given a diagram, the student understands that different fractions can represent the same magnitude.	i. Is not able to generate equivalent fractions without being given a diagram.
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4.3	i. The student can generate simple equivalent fractions using a visual model (i.e., area model or number line). ii. The student can find common denominators needed to write equivalent fractions i.e. $\frac{3}{4}$ as $\frac{18}{24}$.	i. The student confuses relative equivalence and absolute equivalence. The fractional representation may be equivalent but the value is not equivalent (i.e., $\frac{1}{4}$ of a meter is not the same distance as $\frac{3}{12}$ of a kilometer). ii. Cannot generalize the process that dividing the denominator into "n" equal parts results in a numerator that is exactly "n" times as big.

Example of Sublevels

Equivalent Fractions Progression

	Level Description	Misconceptions
4.1	i. Given a diagram, the student understands that different fractions can represent the same magnitude.	i. Is not able to generate equivalent fractions without being given a diagram.
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4.4	ii. The student understands the mathematical reasoning behind generating equivalent fractions ($n/n * a/b = a/b$), including that a number divided by itself is 1 ($n/n = 1$), and the identity property of multiplication ($n * 1 = n$). The student can generalize the dividing the denominator into "n" equal parts results in numerator that is exactly "n" times as big.	

MStar Process

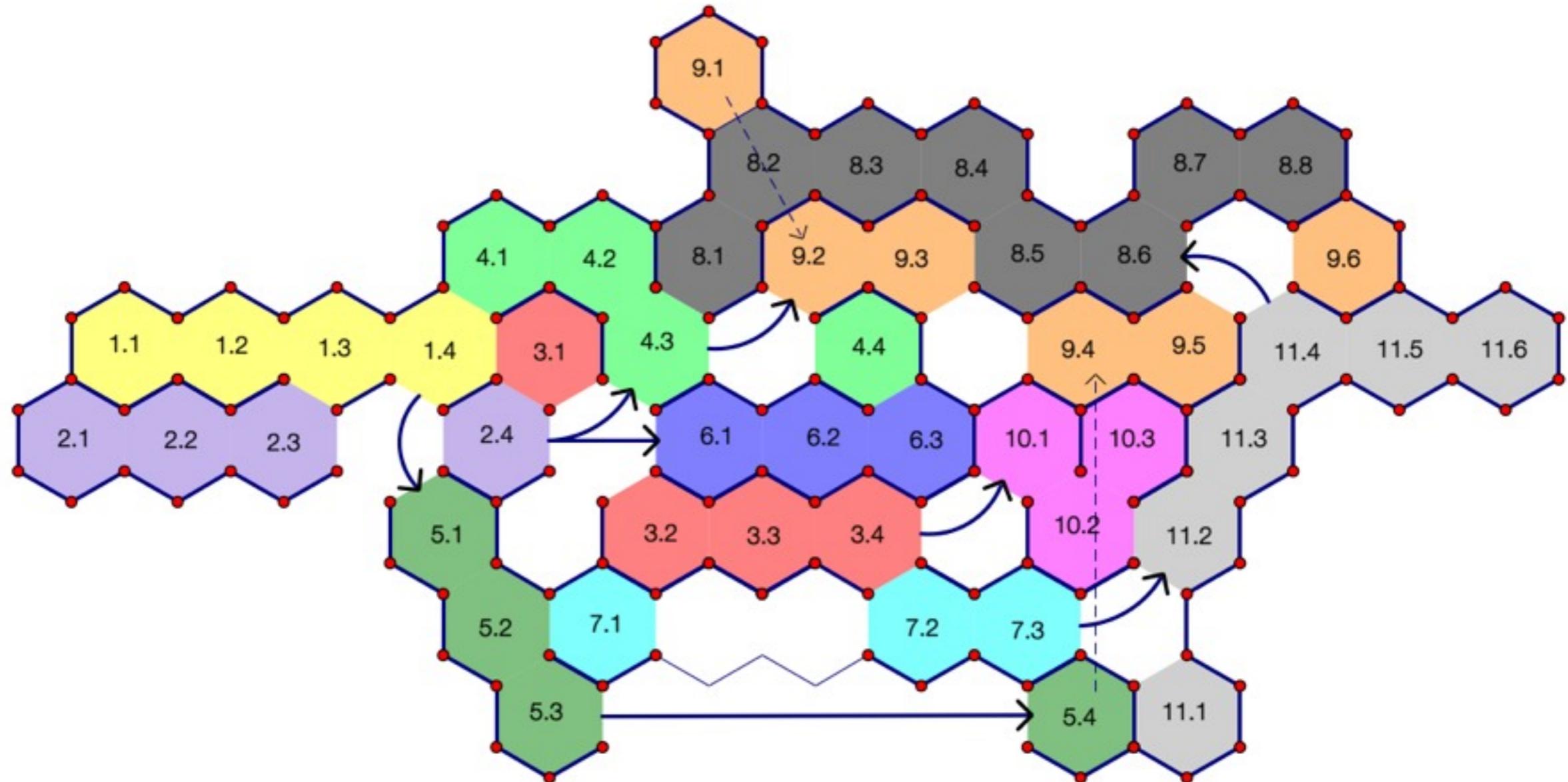
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Interaction of Progress Variables: LP1

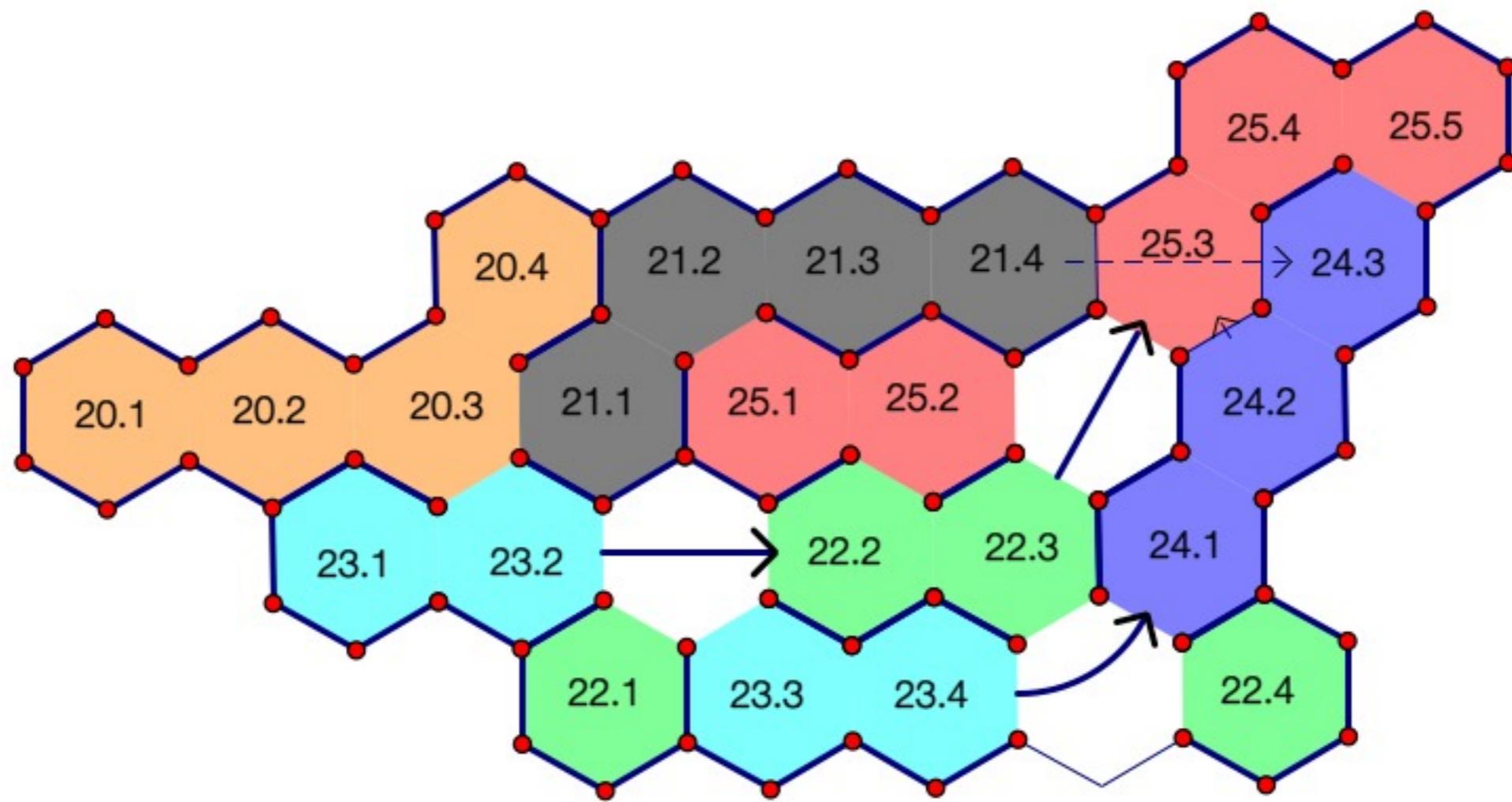
Interaction of Progress Variables: LP1

LP1: Understanding Positive Rational Numbers, their Representations, and their Uses



Interaction of Progress Variables: LP2

LP2: Understanding Variable Expressions, and their Applications



Validity

- Qualitative analysis from student interviews
- Understanding how these can be used at a “systems” level for content