Rethinking Professional Development for Elementary Mathematics Teachers

By Erica N. Walker

Introduction

Researchers have found that despite reformers' best efforts, teachers' mathematics classroom practice remains largely unchanged—in part because teachers hold fast to their own mathematics understandings, attitudes, and experiences (Ball, 1996; Raymond, 1997; Tzur, Martin, Heinz, & Kinzel, 2001). In particular, in the last decade, elementary mathematics teachers have found themselves balancing a number of sometimes competing requirements in their teaching: adhering to mathematics reform initiatives in their school, district, and/or state; meeting the expectations of principals and parents; and finding ways to ensure that their students are able to perform adequately on standardized tests that have significant ramifications for teachers and students if students fail (Manouchehri, 1997; Raymond, 1997; Schoenfeld, 2002). In recent years, many teacher education programs have begun to address elementary mathematics instruction by helping prospective elementary teachers expand their knowledge of math-

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ematics content. This has often occurred through mandating more mathematics courses (American Mathematical Society, 2001); but often these courses have not focused on the special needs of elementary teachers.

Further, the support that these teachers receive once they leave teacher education programs is often sporadic and shallow (Borman & Associates, 2005). With the advent of new curricula, professional development for elementary teachers is often heavily focused on implementation of a particular curricular package, which may target organizational or logistical requirements of the curriculum rather than mathematics content or pedagogy aligned with content objectives. Many of these curricula, seeking alignment with National Council of Teachers of Mathematics (NCTM) standards reform documents (1989; 2001), require substantially more teacher engagement with students than 'traditional' textbooks and their framers expect elementary teachers to deeply understand the underpinnings of elementary mathematics (D'Ambrosio, Boone, & Harkness, 2004). In order to spur student learning of mathematics, rather than just performance, teachers are expected to respond to student misconceptions, help students develop conceptual understanding, and provide multiple curricula and media to do it (American Mathematical Society, 2001; Frykholm, 1999). This can be difficult when teachers themselves may hold misconceptions, have limited rather than deep conceptual understanding of mathematical topics, and may not understand how working with different media and manipulatives can contribute to student thinking and learning in mathematics.

I developed a professional development model designed to address these issues as part of a larger study of an intervention, Dynamic Pedagogy,[1] targeting Grade 3 students and their teachers in an ethnically and socioeconomically diverse school district in upstate New York. My fellow researchers and I, through Dynamic Pedagogy, sought to improve student learning and performance in mathematics, as well as develop 'habits of mind' conducive to life long learning habits among these children. However, we soon realized that we first had to enhance teacher understanding of mathematics and help teachers to create mathematics classroom experiences that would foster student thinking, so that teachers would be able to effectively implement the Dynamic Pedagogy intervention. This paper discusses the professional development model and describes how it was reflected in the classroom practice of participating teachers. Because much of the literature in teacher education is silent on the mechanisms by which teacher education and professional development affect actual classroom practice, I also report how this model influenced one teacher's planning and instruction in mathematics.

Background

> Teaching elementary mathematics requires both considerable mathematical knowledge and a wide range of pedagogical skills. For example, teachers must have the patience to listen for, as well as the ability to hear the sense… in children's mathematical ideas. They need to see the topics they teach as embedded in rich networks of

> interrelated concepts, know where, within those networks, to situate the tasks they set their students and the ideas these tasks elicit. In preparing a lesson, they must be able to appraise and select appropriate activities, and choose representations that will bring into focus the mathematics on the agenda. Then, in the flow of the lesson, they must instantly decide which among the alternative courses of action open to them will sustain productive discussion. (American Mathematical Society, 2001, p. 55)

The American Mathematical Society and Mathematical Association of America's (2001) emphasis on mathematical and pedagogical knowledge for elementary teachers has been underscored by the research literature documenting that elementary mathematics teaching and learning in the United States has too often been limited to rote, didactic experiences that do not propel mathematics thinking (Ball, 1996; National Research Council, 2001). Indeed, "[w]e must recognize that many current elementary teachers' mathematical understanding is far from ideal (Ma, 1999)" (Farmer, 2003, p.333). Many elementary mathematics teachers "were not adequately prepared by the mathematics instruction they received" (AMS, 2001. p. 55). They may have limited content knowledge and in addition, may have an aversion to exploring mathematics content more deeply (D'Ambrosio, Boone, & Harkness, 2004; Frykholm, 1999; Thompson, 1992). This is a critical aspect, given that "subject matter knowledge significantly impacts classroom instruction as well as teachers' decisions with respect to the selection and structure of teaching content, classroom activities, assignments, and choices in curriculum materials" (Shulman & Grossman, 1988, p. 3). Thus, it should be no surprise that professional development that focuses on the enhancement of content knowledge is linked to improvement in student mathematics achievement (Saxe, Gearhart, & Nasir, 2001).

Good professional development for elementary teachers should effectively address this problem of enhancing teachers' content knowledge. Content knowledge, however, is not enough: it is important that teachers develop effective teaching strategies and practice (Graham & Fennell, 2001). In doing so, they should relinquish their own reliance on procedural explanations for mathematics concepts[2] (Frykholm, 1999). Echoing the recommendations of NCTM and AMS, Frykholm (1999) urges that "[t]eachers must understand mathematics deeply themselves if they are to facilitate the types of discussions and handle the various questions that emerge when learners are engaging in authentic mathematical experiences"(p. 3).

However, it must be noted that elementary mathematics content development for teachers is hampered by the perception by many in the field that elementary mathematics is 'simple' and not worthy of extensive discussion, despite evidence of its rich conceptual underpinnings (Frykholm, 1999; AMS, 2001). Much of the understanding that both elementary and secondary teachers say they have is procedural and rule-based, rather than conceptually oriented. This understanding colors teachers' views of instruction and limits their pedagogy to lecture and explication of procedures rather than expanding it to include students' thinking and ideas and development of conceptual knowledge and understanding.

The idea that teachers' content knowledge is linked to their pedagogical practices is not a new one. Shulman and Grossman (1988) have written extensively about the relationships between content knowledge, pedagogical knowledge, and pedagogical content knowledge. In particular, pedagogical knowledge can be discussed and shared, but is more powerfully "shaped through experiences with children" (Graham and Fennell, 2001). Pedagogical content knowledge (PCK) incorporates content knowledge of a discipline but also teachers' "knowledge of the subject, supplemented with knowledge of students and of learning; knowledge of curriculum and school context; and knowledge of teaching" (Manouchehri, 1997, p. 201). Specifically, PCK requires teachers to understand "multiple representations of mathematical ideas, topics, and problems; the complexities of teaching and learning certain mathematical concepts; and the cognitive obstacles that learners face when they engage in certain topics in the mathematical curriculum" (Manouchehri, 1997, p. 201).

This third category of knowledge reflects the importance of selecting and using appropriate activities and strategies in the classroom based on teachers' understanding of what students know and need to learn. In short, teachers should know that it is not sufficient to just use pattern blocks[3] as representations of fractions in a lesson and have students complete a worksheet about fractional parts; rather, teachers should know how and why these blocks could be used to develop students' understanding of fractional concepts.

Yet we see less developed research about this important but practical aspect of teaching—how teachers select activities appropriate for what their students need to learn (see Stein, Smith, Henningsen, and Silver (2001) for an exception). Further, what do teachers understand about their own knowledge and, as importantly, how do they implement this understanding in their classrooms? One way to discern teachers' understanding of mathematics and their instructional practice is to analyze the kinds of tasks that teachers provide and the kinds of questions that teachers ask students during mathematics lessons (Tirosh, 2000). As Sullivan and Clarke (1991) state:

> Good questions have more than one correct mathematical answer; require more than recall of a fact or reproduction of a skill; are designed so that all students can make a start; assist students to learn in the process of solving the problem; and support teachers in learning about students' understanding of mathematics from observing/reading solutions (p. 337).

But this level of questioning may be very different from the types of experiences that elementary teachers received themselves—in elementary school, secondary school and post secondary education institutions (AMS, 2001). Improving elementary mathematics instruction, then, requires that we consider that the extent of mathematics preparation for teaching for many elementary teachers was attained in teacher preparation programs where they may have taken a single mathematics methods class. The role of Professional development becomes critical—and should be used as a site for content knowledge development.

In summary, effective professional development models should respect and address teachers' existing beliefs about mathematics because these affect their instruction. Often this is a hidden agenda of providers. It should be noted, however, that teachers may not choose to participate in a professional development project to change their own beliefs about mathematics and its teaching, but this is often the "hoped-for outcome on the part of providers" (Farmer, 2003, p. 332).

Professional development should also address the links between content knowledge, pedagogical knowledge, and pedagogical content knowledge (Shulman & Grossman, 1988; Ball & Bass, 2000). This underscores the need for professional development to be situated in school- and classroom-based contexts.

Professional development should be sustained and ongoing. As Ma (1999) notes, "a single [PD] workshop, without periods of gestation and sustained support, does not afford sufficient time for teachers to develop a deep understanding of the mathematics they teach; such understandings develop, if at all, over longer periods" (p.333).

Finally, but most importantly, I argue that it is necessary for teachers to be active participants in and constructors of the professional development experience so that they can share and analyze their own rich classroom experiences.[4] In short, teachers should participate in the knowledge construction process that, hopefully, emerges from professional development. In designing the professional development experience for teachers I asked, "How do we create opportunities for teachers to engage in collaboration that facilitates their own development—so that they can effectively model this important aspect of mathematics learning for students?" As we encourage opportunities for students of all ages to work together as communities of learners—a recognized effective teaching strategy—it is important that we create these communities for teachers as well (Lachance & Confrey, 2003).

Method

Designing the Professional Development Model

Professional development for teachers was one major component of the Dynamic Pedagogy intervention. Our initial goals for professional development targeted three interrelated major problems affecting elementary school teaching: inflexible attitudes about mathematics and its teaching (Raymond, 1997), lack of deep understanding of basic mathematical concepts (Ma, 1999), and a teacher-centered approach to teaching that does not utilize students' substantial knowledge of mathematics, gained from their in- and out-of school experiences (Walker, 2003). However, the primary goal was to address what the framers of Dynamic Pedagogy believed to be an underlying fundamental element: to enhance teachers' mathematics content knowledge meaningfully.

In this section, I focus on sharing the process of developing and revamping the professional development model. The initial focus shifted to one that incorporated a framework to help teachers more effectively design and implement classroom activities that demonstrate connections across mathematics topics; provide students with multiple representations of mathematics concepts to facilitate students' mathematical knowledge development; and address student misconceptions of mathematics topics. These three components (connections, representations, and misconceptions) eventually formed the core of the professional development model, referred to as CRM (Figure 1). As the professional development seminars progressed, the research team expected that CRM professional development would also aid teachers in selecting appropriate curricular directions and providing students with rich problem-solving opportunities. In essence, I wanted to ensure that the CRM professional development framework, focusing on mathematics content, would help teachers better implement Dynamic Pedagogy in their classrooms.

To address teachers' mathematics knowledge and pedagogy, I designed and, along with co-facilitators, held monthly professional development seminars (Table 1) with nine grade 3 teachers participating in the Dynamic Pedagogy intervention. Seminars included four two and a half-hour after-school sessions, and two full day workshops. In addition, my co-facilitators and I held a three day Summer Institute the summer preceding the implementation of Dynamic Pedagogy. Each seminar targeted a particular elementary mathematics topic (e.g., fractions, number sense, geometry).

Because the research team was interested in the learning that took place in professional development seminars and also evidence of teachers using what they had learned during class instruction, we collected data from multiple sources. During each seminar, held in September, November, February, and April, members of the research team took field notes, documenting problem solving and pedagogical discussions. Members of the research team observed teachers' lessons during

Figure 1. CRM (Connections, Representations, and Misconceptions) Professional Development Model.

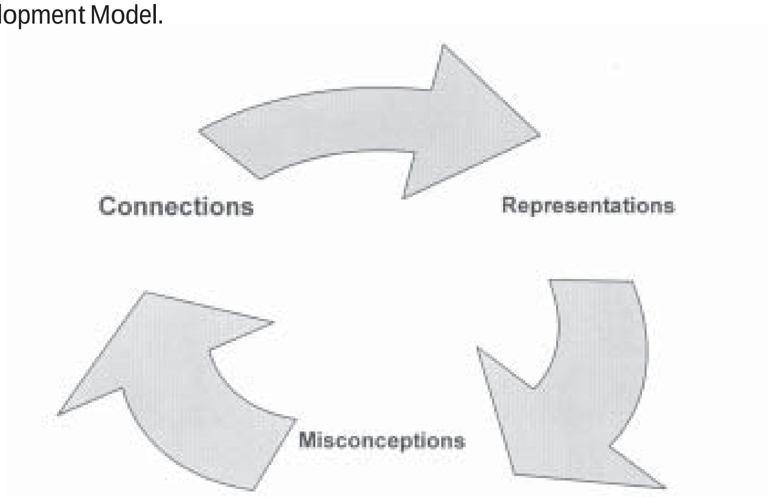


Table 1. Description of Professional Development Seminars

Date	Content Focus	Duration
June 2003	Dynamic Pedagogy Orientation; Number Sense; Fractions; Geometry; Measurement	3 days
September 2003	Number Sense	2.5 hours
October 2003	Dynamic Pedagogy Principles; Number Sense; Fractions	1 day
November 2003	Fractions	2.5 hours
January 2004	Dynamic Pedagogy Principles; Fractions and Geometry	1 day
February 2004	Geometry	2.5 hours
April 2004	Measurement	2.5 hours
May 2004	Dynamic Pedagogy Principles; Reflection	2 hours

each of the four Dynamic Pedagogy units (number sense, fractions, geometry, and measurement), and videotaped a teacher's lesson at least once. Field notes (Denzin & Lincoln, 2000) were taken during observations. After observing lessons, researchers conducted informal interviews with teachers and provided critical feedback. Teachers submitted portfolios for each unit, which included lesson preplans, plans, self-assessments, and samples of student work. Because the unit on fractions took place several months after the first unit, number sense, we were able to document evidence of the impact of professional development seminars. Later in this paper, I focus specifically on evidence of one teacher who incorporated elements of the CRM framework in her teaching as the year progressed.

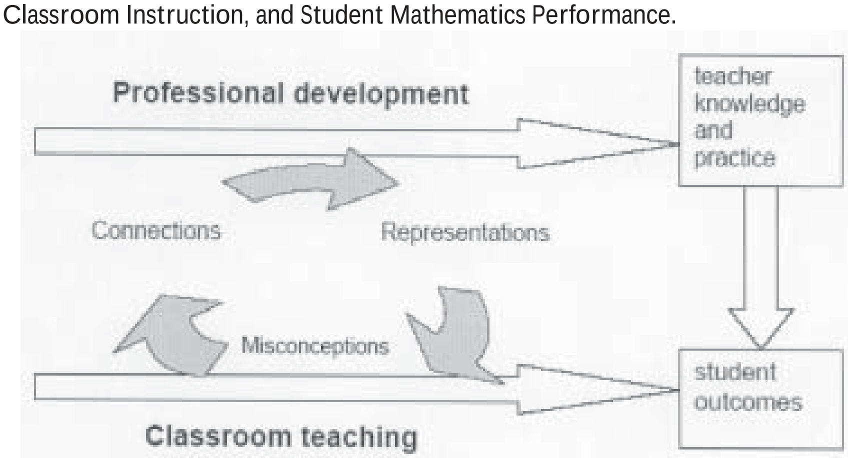
Developing the CRM Framework

The CRM framework for professional development emerged, in part, because of the major focus of the professional development seminars, which was to develop teachers' content and pedagogical content knowledge. Initially, we (the facilitators) began professional development sessions with teachers solving mathematics problems collaboratively. Although the problems were not at a level beyond high school algebra, we found that teachers were visibly anxious and reluctant to work together. Comments like "This is too hard," "I forgot how to do this," and "I was never good in math" abounded. Even though the problems teachers were asked by seminar leaders to solve were linked to the content of the curricular units that teachers were engaged in at those points in time with their students, the sessions could not progress in the ways we wanted them to without addressing teachers' perspectives about mathematics (Raymond, 1997).

I realized that we needed to "work from what teachers *do* know-the mathematical ideas they hold, the skills they possess, and the contexts in which these are understood" (AMS, 2001, p. 57). I adjusted the organization of the sessions to address teachers' strengths. While still maintaining a commitment to developing and expanding teachers' mathematical knowledge, my co-facilitators and I did so by integrating discussions about problem solving, mathematics content, and pedagogical practice. We did this by organizing the professional development sessions so that teachers and seminar leaders talked about content as a part of discussions about teaching and learning episodes, student work, and lesson planning.

I developed the CRM professional development framework based on the research literature and the recommendations of the NCTM and AMS pertaining to effective teaching and staff development for discussing teaching and learning episodes, student work, and lesson planning. The three major components of the framework—connections (between mathematical topics, students' in- and out-of-school knowledge, and procedural and conceptual knowledge); multiple representations (of mathematical concepts, problems, and solutions), and misconceptions (about mathematics concepts and procedures)—do not operate in isolation (Figure 2). Rather, I expect that these elements inform each other during the teaching-learning process. The teachers and facilitators thought and talked at length about students' misconceptions, and how to help them develop multiple representations of concepts and see connections between mathematical topics. We worked to establish a space for teachers to engage in more rigorous mathematics, as well as develop their pedagogical knowledge. Because teachers worked collaboratively and heard from other teachers about their work, they were able to benefit from the experiences of others. Essentially, we attempted to model with the teachers during professional development sessions

Figure 2. Hypothesized Relationships between CRM Professional Development, Classroom Instruction, and Student Mathematics Performance.



the way that we wanted them to help their students to engage in mathematics thinking and learning in the classroom. Like Farmer (2003),

> . . . [b]y using activities that were adaptable to the elementary level and sufficiently challenging to the teachers, we could address the desire of some teachers to add to their repertoire while still meeting a need to learn more mathematics. (p.333-334)

My fellow researchers and I hypothesized that the work that teachers did during professional development seminars would inform their classroom teaching; similarly, their classroom teaching experiences would inform the content of the professional development seminars. This iterative and reciprocal process, we hypothesized, would enhance teachers' teaching and could also help to spur student achievement in mathematics.

We developed a coding scheme to analyze the presence of CRM indicators in teachers' instructional practice. A team of two raters reviewed transcripts of videotaped lessons for the fractions units, and coded teachers' questions posed to students, conversations with students, and responses to student questions (teacher-learner interactions) as encompassing connections, representations, misconceptions, or procedures. For example, raters coded the following:

> Teacher: We have been working with our *fraction strips and our pattern blocks* to study fractions [representations].
>
> I want to start out by talking about a little bit of real life experience. When do you use fractions *in your everyday life*? [connection-real world]

Very rarely did coders have different evaluations of the episodes; when this occurred it was usually because a particular exchange encompassed more than one CRM category. In these instances coders discussed the teacher-learner interactions and came to an agreement about how the episode should be coded. Some episodes, underscoring the interaction between the elements of CRM, belonged to more than one category and were coded accordingly. Following coding, raters counted the number of teacher-learner interactions within the lessons that revealed teachers' emphasis on connections between mathematical topics, multiple representations, and/or student misconceptions. In addition, we found there were certain tasks that solely required students to recall a certain procedure, without making connections to conceptual understanding (Table 2).

Findings

Learning During Professional Development Seminars

I first describe below how elements of the CRM model were enacted in a sample professional development seminar focusing on fractions (Table 1). I began by facilitating a discussion about participants' prior classroom experiences teaching

Table 2. CRM Categorization of Representative Teacher Tasks and Questions.

<table>
<tr><th>Procedures</th><th>Connections</th><th>Representations</th><th>Misconceptions</th></tr>
<tr>
<td>Here are some Hearts
• Color in a few of them.
• Write the fraction for the amount that is colored.

Use your Fraction pieces to help you solve the following problems:

| 1/3 | 1/3 | =_?__

(2/3 of a 1-strip = 4/6 of a 1-strip)</td>
<td>Ms. D: "Can we think of equivalent names for the number 6?"
Student Responses:
"3 + 3 = 6"
"3 x 2 = 6"
"12 divided by 2 = 6"
Ms. D.: So there are also different ways to represent fractions. Some fractions can measure the same amount, but they have different 'names'. They are equivalent but we use different numbers to represent them. Let's try to find some fractions that are equivalent to ½.</td>
<td>Money can be represented in different forms. Complete the chart to show the different forms.

<table><tr><th>Money</th><th>Decimal Form</th><th>Fractional Form</th></tr><tr><td>1 dime</td><td>$0.10</td><td>10/100</td></tr><tr><td>20 pennies</td><td></td><td></td></tr><tr><td>3 dimes</td><td></td><td></td></tr><tr><td>2 dollars, 3 dimes, 1 nickel.</td><td></td><td></td></tr></table></td>
<td>Mr. S and Ms. T ordered the same size pizza for their families. Mr. S ate 9/12 of his family's pizza and Ms. T ate ¾ of her family's pizza. Who ate more? Explain your answer.

[Ms. M and her students are discussing the sharing of a graham cracker among four people]
Ms. M: Suppose I wanted to share this cracker with my friend.
Student1: Give them ½ of the cracker.
Ms. M: But I thought it was in fourths.
Student2: They get 2/4s
Ms. M: I don't understand what she means. Who can explain?
Student 3: Out of 4 pieces you have 2.
Ms. M: Does that mean that I have more than I had before? Students: [Yes][No][They're equal]</td>
</tr>
</table>

students about fractions. Teachers talked together about how they have introduced fractions to students and their rationale for introducing them in this way. The teachers' ideas shared during this particular part of the session encompassed primarily activities (games or tasks to do with students) and questions they ask of students:

> Gloria: I ask my kids if they know what a fraction is. But they just came back with "numerator" and "denominator." They don't really understand what those words mean.

Other teachers agreed with Gloria's assessment. The discussion then turned to other common misconceptions that children have or mistakes their students made in working with fractions. Like Tirosh (2000), my co-facilitators and I found that teachers initially focused on procedural or algorithmic aspects of working with fractions.

I then presented a problem from the 1992 National Assessment of Educational Progress mathematics assessment for 4th graders:

> Think carefully about the following question. Write a complete answer. You may use drawings, words, or numbers to explain your answer. Be sure to show all of your work.
>
> José ate ½ of a pizza.
>
> Ella ate ½ of another pizza.

> José said that he ate more pizza than Ella, but Ella said they both ate the same amount. Use words and pictures to show that José could be right. (Dossey, Mullis, & Jones, 1993)

This problem, and the ensuing discussion about the fact that only about a quarter of fourth graders in the US gave a satisfactory or better response to the problem (Dossey, Mullis, & Jones, 1993), allowed for rich dialogue to develop. Teachers discussed the nature of the problem, and mentioned that too often students and teachers begin to overemphasize that "one half is always equal to one half." They talked about the importance of considering the "size of the whole" when talking about this problem. Although the teachers were not instructed to use the problem in their lessons, four teachers modified it and used it in some way.

The discussion during the fractions seminar also served to elicit misconceptions that teachers themselves may hold about fractions. Initially, the discussion focused on procedural mistakes that students make with terminology (confusing the numerator and denominator, for example) and notation (not understanding how to write a fraction; e.g., writing ¾ as 3_4). However, teachers moved fairly quickly to discussing conceptual problems that children have with fractions, citing examples from their own teaching experience. Some of their ideas about students' misconceptions included the lack of understanding that a fraction is a part of a whole, and that in some ways fractions do not operate like whole numbers. The notion lingering from whole numbers that the larger the number the bigger it is, especially with equivalent fractions, was deemed problematic by teachers. For example, 3/4 = 9/12, but many students think the second fraction is bigger because it has larger numbers. The facilitator then discussed how these ideas often stem from children's extensive experience with whole numbers—thus, they think that fractions should operate in the same way (Smith, 2002).

I then asked teachers to consider how they or their students might describe a fraction. Teachers' responses opened the door for the facilitators to point out that their comments (about "½ of a set of crayons" or "½ of a cake") describe the various ways that fractions can be represented—via the set model or unit model (National Research Council, 2001). Asked if there were any other ways to represent fractions, teachers then talked about a number line. In constructing the discussion in this way, the facilitator was modeling for teachers that others' contributing ideas could help to shape a lesson or instructional activity and develop deeper learning (Tirosh, 2000). Similarly, teachers could use their students' own prior experiences to think about how they could develop lessons that were meaningful to students and built on the knowledge that students bring with them to the classroom. Providing students with exposure to these multiple representations of fractions could help to break down misconceptions and solidify student conceptual understanding.

Finally, I asked teachers to also consider the ways in which fractions were connected to or used to develop other mathematical topics. Teachers talked about the commonly understood links between fractions, decimals, and percents. In addition, they raised the points that fractions are important components of measurement (measuring to the 1/8th of an inch with a ruler or that a ‘quarter to 12’ is 15 minutes to 12 Noon because 15/60 is a quarter of an hour).

After this discussion, the teachers worked together on mathematics activities that underscored the multiple representations of, misconceptions about, and connections of fractions to other topics. Many of the teachers had worked with pattern blocks before, but one of the activities for the day was to reclaim these as area models of fractions and to consider misconceptions of students about representing particular regions of the blocks as fractions. In addition, professional development leaders and teachers manipulated the idea of ‘whole’ through several activities with: pattern blocks (where, for example, 2 pattern blocks or portions of pattern blocks served as the ‘whole’; relative size (analyzing student responses to a question about ‘who had more pizza’); and fraction strips (where they determined that the fractional portions of a whole created by folding a piece of rectangular paper in half repeatedly is yielded by the function $[1/2]^n$, where n is the number of folds).

The work accomplished in professional development seminars is aligned with the process that teachers are asked to use for lesson planning while participating in Dynamic Pedagogy. The planning process consists of *preplanning*, *planning*, and *reflection.* We expect this process to be iterative, in that during planning and teaching teachers may have to make adjustments to their intended plans based on circumstances that arise when engaged in the actual teaching process (Armour-Thomas, Gordon, Walker, & Hurley, 2002). The opportunity to evaluate the elements of preplanning and planning occurs during the teaching phase and during the reflection phase. While these are important elements of Dynamic Pedagogy lesson planning, one of our key questions is how our CRM professional development model is reflected in teachers’ planning and practice. How do teachers apply what they have learned in professional development seminars to their actual practice? Are teachers providing multiple representations of mathematical problems and concepts, opportunities for students to work through their misconceptions, and linking mathematics topics and students’ prior in- and out-of-school experiences in mathematics?

Teachers’ Use of CRM in Instruction

The next section of this paper describes, analyzes, and evaluates the opportunities that teachers provided in their classes during a unit on fractions for students to engage in mathematical thinking and problem-solving. In addition, I will focus on one teacher’s preplanning, planning, and practice for a fractions lesson. These examples are not necessarily exemplary, but rather illustrative of how the CRM framework can have an impact on classroom teachers’ lesson planning and instructional decisions.

Table 2 provides examples of tasks provided and questions asked by teachers during a “fractions unit.” These selected questions and tasks reveal that teachers have a variety of ways of addressing connections, multiple representations, and misconceptions in their work with students. The procedures category shows, for comparison, the types of questions that teachers ask students that reinforce certain mathematics algorithms or procedures. Although the activities may be motivating for 3rd grade students (coloring in hearts or flowers; manipulating fraction strips), these tasks do not ask students to engage in a complex way with the material—they are not necessarily solving meaningful problems (Stein, Smith, Henningsen, & Silver, 2000).

However, the tasks teachers provided in the connections, representations, and misconceptions (CRM) categories reveal a variety of levels of their thinking and selection within the lesson. In her lesson about equivalent fractions, Ms. D. asks students to think of the multiple ways we use equivalence in mathematics and the 'real world.' In doing so, she helps students make connections between mathematics and 'real world' experiences, as well as link the idea of equivalence to fractions and integers. This grasp of equivalence is particularly important when students begin to engage in algebraic thinking in the later grades. In asking students to think about whole numbers as well as fractions, she is making connections between two (or more) mathematics topics that will be useful to supporting student thinking about number.

Of most interest to the researchers is how teachers address what they find in their experience to be common student misconceptions. Following our professional development seminar focusing on fractions, five of the nine teachers in this study incorporated one common student misconception about fractions in their lessons. I present two of them in Table 2.

Mr. V and Ms. W. teach their 3rd grade classes together as a team. The question posed in Table 1 opened their lesson:

> Mr. V and Ms. W ordered the same size pizza for their families. Mr. V ate 9/12 of his family's pizza and Ms. W ate 3/4 of her family's pizza. Who ate more? Explain your answer.

They used this question as a motivating question without telling students how to think about this problem. Because this lesson was videotaped, we could see the process in which students engaged. Several students could be seen drawing, or taking out their fraction strips to answer this problem.

Important for Mr. V and Ms. W's instructional purposes was the caveat that the pizza they both ordered was the same size pizza. This arose in a professional development seminar discussion: the fact that ½ is not always equal to ½, and that the size of the fraction depends on the size of the whole, or set, to be measured. Their construction of the task in this way reveals important links to their own pedagogical content knowledge (Shulman & Grossman, 1988).

From Table 2, Ms. M's question about the graham crackers was also used to motivate an activity with equivalent fractions. What is interesting about Ms. M's class is that the students are in disagreement about whether or not 2/4 is more than ½, again illustrating a point raised in the professional development seminar focusing on fractions. To resolve this misapplication of the size of whole numbers to fractions, Ms. M asked the students to compare the size of the sets of broken graham crackers and to evaluate which set was larger. This underscores the concept of equivalence for students.

While it is illuminating to examine elements of teacher practice across several teachers, we were very interested in how teachers implement CRM elements throughout the instructional process. I turn now to an examination of how one teacher enacted the CRM framework throughout her preplanning, planning, and practice.

Throughout the implementation of the Dynamic Pedagogy intervention, we noted that one of the participating teachers, Davinia,[6] incorporated more and more elements of the CRM model in her teaching. Visitors to her class were impressed with the ways in which students actively participated and seemed to be engaged in their own learning. I focus on Davinia here to evaluate her practice during the unit; her work serves as what we consider to be exemplary implementation of the CRM model.

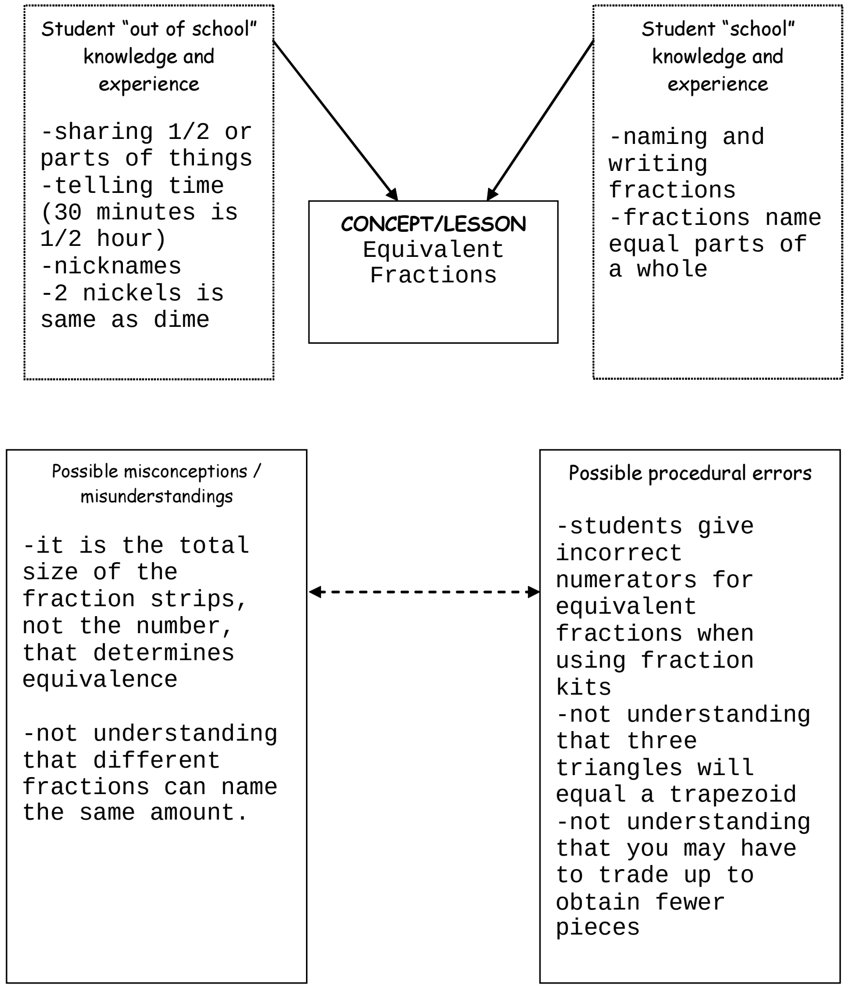
Davinia's Planning Process

As aforementioned, we asked teachers to engage in *preplanning*. The preplanning template in Figure 3, which I designed, shows the ways in which I wanted to help organize teachers' thinking about any one particular mathematics topic. I provided space for teachers to think about how students' school and out of school knowledge and experience relate to a particular topic, the misconceptions/misunderstanding that students might have before, during, and after working on a particular topic, and the possible errors procedural errors that students might make. Davinia's preplan (Figure 4), completed several weeks after the professional development seminar targeting fractions, shows that she was thinking about students' potential 'out of school' experience relevant to fractions (Armour-Thomas, et al, 2002). She revisits some of the themes that she and her fellow teachers discussed during that session—namely, the experiences that students have with sharing and telling time. In addition, she describes some out of school experiences that students may have with the notion of equivalence, e.g. nicknames and money.

Davinia's ideas about the possible misconceptions students hold and errors that students may make about fractions reveal that, in her mind, some of these are tied closely to the activity that she plans to have students do with fraction strips or pattern blocks. The second misconception she has listed is related to a conversation from our professional development seminar—that "different fractions can name the same amount."

Many of these preplan ideas emerged in various sections of the actual lesson plan. Although Davinia's goal is to have students use concrete materials to find equivalent fractions, it is unclear exactly what she wants her students to be able to do by the close of the lesson. However, her ideas about using nicknames to introduce 'equivalence' and addressing students' misconceptions about size and equivalence through a closing activity appeared in the preplan.

Figure 3. Davinia's Preplanning Template



Davinia's Practice

We analyzed Davinia's videotape and transcript of this lesson on fractions to determine if her intended plans were carried out in the lesson and also to analyze her "teacher talk" with students, around connections, representations, and misconceptions. There were several instances during Davinia's lesson when she helped students to think about connections between fractions and their lives, as well as connections between mathematical terms and everyday terms.

Figure 4. Davinia's Lesson Plan.

LESSON PLAN

Goals for the unit:
To use concrete materials to find equivalent fractions.

Objectives of the lesson:
-to use fraction kits and pattern blocks to find equivalent fractions
-to identify fraction parts
-to exchange equivalent fractions

Materials:
-fraction kits
-overhead projector
-spinners with 1/2, 1/3, 1/4,2/3,1/6,5/6
-pattern blocks (without tan or orange)
-hexagon worksheet
-yellow crayons or markers

LESSON PHASES:

Initiation:
-Introduce equivalence by teacher saying that some people are known by a different name (ex. Jimmy for James)

LESSON PHASES *continued*:

Development:
(1) Take out fraction strips. Find which fractions are equivalent to: 1/2 (2/4, 4/8, 8/16); 1/4 (2/8, 4/16)

(2) Introduce 'Fraction Cookie Game' to class. Students are to collect pieces of pattern blocks to build hexagon. Pairs of players take turns spinning spinner and adding to their cookies. Make trades wherever possible to obtain the fewest number of pieces. Winner is the person to have completed more hexagon cookies (play in pairs)

Closure:
Word problem to assess knowledge: Brian ate 2/8 of a pizza pie. Michelle ate 1/4 of the pie. Brian says that he ate more. Michelle claims that they ate the same amount. Who is right? Explain. You may draw a picture.

"Let's think…If someone has a name, let's say James, but people also call him Jimmy, is he the same person?"

"We talked about the number on the clock as 12 pm, and we called it noon… " "Another way to think of six is as 3+3…what are other ways?"

"All of these are different ways of representing the same amounts . . ."

"Let's talk about fractions and see if we can figure out what equivalent fractions are."

"So now we are making fractions that are the same size but have different names."

In addition, she provided opportunities for students to engage in thinking about fractions through multiple representations of this concept.

Davinia: Look at your fraction strips—who can tell me what other fractions would be equivalent to ½?

Keith: 4/8

Davinia: Let's see if he is right, Keith, come up and show us. [Davinia places the new fraction strip on the blackboard; Keith comes up to the blackboard to explain. He is allowed to change the positioning of the strip in order to present his explanation.]

Davinia: Who has another way to make ½? [Most of the students raise their hands.]

Art: I used 2/4. [*Art* comes up to the blackboard to demonstrate that the fractions are equal.]

Davinia: Are we making fractions of the same size that have different names?

Davinia: [Using pattern blocks] So if I exchange the trapezoid for those green, what am I saying?

Students: 3/6 are equivalent to 1/2.

Although throughout the lesson, Davinia interspersed attention to students' emerging, re-emerging, and/or lingering misconceptions while working with the fraction strips and pattern blocks, her planned closing activity reinforced for students what the teachers during professional development had discussed and what she had mentioned on the preplan. With this question, she is asking students to generalize from concrete materials to see if her lesson on equivalence was successful: do students understand how equivalent fractions, an important building block for future computation and algebraic problem solving, operate? Her closing question to the students about who ate more of a pizza pie relates to her thinking about how to get 3rd graders to understand that 2/8 of a pizza is not more than 1/4 of the same pizza:

> Brian ate 2/8 of a pizza pie. Michelle ate 1/4 of the pie. Brian says that he ate more. Michelle claims that they ate the same amount. Who is right? Explain. You may draw a picture.

Students' work on this problem reveals that they use a variety of strategies to answer the problem. For example, Jane7 wrote:

> Michelle is right because I mesured [sic] 2/8 and 1/4 , and I found out that 2/8 is the same as ¼. I found that out because I put one fraction strip that said ¼ under another fraction strip that had 2/8, and that how I knew they were equivalent.

Lance wrote:

> Michelle is right because when I crossed multiply I found out the fractions were equal.

Joy wrote:

> Michelle is right because 1/4 is the same as 2/8 just like one eighth is half of one forth. Then if one eighth is half of one forth [sic] then two eighths must be one fourth.

And finally, Chris wrote:

> Michelle is right because when I put my fraction strips together I relized [sic] that 2/8 is equal to ¼. I also relized Michelle was right by drawing two pies the same size, one with four pieces one with eight pieces. When I colored the right amount in I realized they were the same amount just like Michelle had said. They are equivalent fractions.

Although it is clear that students have various levels of understanding and various strategies of solving the problem, they all have arrived at the correct answer. Lance uses cross multiplication to solve the problem, although Davinia has not explicitly taught this process. Joy's process reveals some sophisticated thinking about fractions as fractional parts of other fractions. Chris uses two methods: his fraction strips and drawing to check his answer. What is unclear is if students like Jane and Chris can expand their thinking beyond fraction strips to answer questions about equivalence without relying on concrete manipulatives.

A few weeks later, Davinia gave her students the following problem:

> There are 18 students in Mrs. Clark's class. 9/18 are wearing sneakers. Half of the class is wearing jeans. Are the same number of children wearing sneakers and jeans? Explain.

This problem elicited some discussion from students. Paul said, "I don't understand the question." There was a chorus from several students: "But it's easy!" Another student chimed in, "But what if someone has on jeans *and* sneakers?"

After agreeing to revise the problem to say that 9/18 are wearing sneakers, not jeans, and half of the class is wearing jeans, but not sneakers, students worked on the problem. Jane wrote:

> Yes, the same number of children are wearing sneakers and jeans because if there are 18 children and half of the class is wearing jeans . . . it's like the same as 9/18. Nine plus nine is eighteen and if you shade 9/18 it is like shade 1/2.

Although Jane has a concrete reference point, she now does not have to actually draw a picture and shade 9 out of 18 children.

Later, Davinia reflected:

> That threw me for a loop when Paul said he didn't understand the problem. He's one of my top students. But then when the other kids talked about it I saw that the way the question was phrased could have been confusing. If I hadn't heard the kids I would have totally missed that.

It is possible that the next time Davinia teaches a lesson incorporating this activity, she will reflect on this experience and use it in her teaching to motivate student discussion.

This work shows that bringing mathematics content to the fore, contextualizing it with classroom experience during professional development, and organizing it using the CRM framework can be useful in helping teachers plan meaningful lessons, select and design thoughtful tasks and activities, and reflect on their practice. Although we are still collecting data from teachers, these results suggest that CRM elements operate together in teachers' teaching and teachers' own thinking about mathematics content. Preliminary data show that teachers may have their own misconceptions that can interfere with their ability to effectively use connections and multiple representations in their teaching.

It is important that professional development activities for elementary teachers meaningfully integrate activities for teachers' use with students (sharing materials, resources, pedagogical ideas) with content. Often professional development for elementary teachers is heavily activity focused—but this may limit the deep mathematical problem solving which could occur if teachers were given activities plus problem solving opportunities themselves to help correct their own misunderstandings and misconceptions.

It is important to note that these teachers all reported high levels of CRM in their practice, but some teachers' videotapes and lesson analyses belie this belief. Thus, this study shows that it is important to analyze and compare teacher talk and practice, because teachers' perceptions may not align with the reality of their classroom. The use of videotaped lessons, student work samples, and teacher reflections are important elements of this professional development model, because all of these 'materials situate the mathematics in context resembling the elementary classrooms in which the subject matter is to be employed" (American Mathematical Society, 2001, p. 94).

Working with teachers to implement a new curriculum or new way of teaching requires that we understand their unique strengths. Different teachers may respond differently to curricular mandates in their classrooms—indeed, our study found that teachers with the same plans may enact them very differently, for reasons having to do with their mathematical content knowledge (Frykholm, 2004), pedagogical content knowledge, attitudes about mathematics and its teaching, and classroom context (Raymond, 1997). In addition, professional development should not be solely driven by logistical and organizational concerns pertaining to the curriculum or instructional strategies in use, but rather should be directly tied to the knowledge needs of students and teachers. Although our data are preliminary, we believe that the CRM professional development model could be used with teachers regardless of the textbook or curriculum in use at their schools.

Finally, too often professional development operates on a deficit model of teachers and their abilities (Borland & Associates, 2005). Teachers often end up being lectured to and shown how to incorporate particular items in their lessons in very procedural ways that do not allow for the complexity of contexts that affect their teaching. By incorporating a model for professional development that presumes that teachers have knowledge and strengths, we can further refine a model of elementary teaching that we are asking teachers to implement: to assume that all students can learn rigorous mathematics, we have to expect that their teachers can have highly developed understanding of both the teaching-learning process and mathematics.

Notes

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[1] Dynamic Pedagogy (DP), an integrated curriculum, instruction, and assessment intervention, seeks to provide elementary students with mathematics experiences that, instead of emphasizing basic skills acquisition, focus on students attaining computational fluency, procedural skill, conceptual understanding, and problem-solving skills (Armour-Thomas, Gordon, Walker & Hurley, 2002; AMS, 1999; NCTM, 1989, 2000). DP's five-pronged theoretical framework comprises Sternberg's triarchic theory of intelligence (1985,1988); Vygotsky's sociocultural perspective of cognitive development (1978); Gordon's (1999) concept of intellective competence; Feuerstein's (1979) mediated teaching-learning experiences; and Artzt and Armour-Thomas's (2001) model of teaching as problem solving. The goal is to improve teacher-learning transactions through curriculum, instruction, and assessment (CIA), with an emphasis on improving teaching-learning experiences for students of color. Major components of the intervention include on-site professional development targeting teachers' planning, implementation, and reflection; the development of mathematics tasks targeted to students' strengths and needs in different but not necessarily disjoint cognitive modalities (creative, practical and analytic tasks) and the effective use of questioning (or probing) in instruction to gauge students' prior knowledge of a mathematics concept and their readiness for new content knowledge; and to activate their interests in mathematics.

[2] Lest we think this problem is limited to elementary teachers, Frykholm reveals that high school teachers may hold misconceptions about elementary mathematics concepts similar to elementary mathematics teachers (Frykholm, 1999).

[3] Pattern blocks are geometric shapes used in elementary education. The set most commonly used in elementary mathematics education includes a regular hexagon, trapezoid (1/2 of the hexagon), equilateral triangle (1/6th of the hexagon), and rhombus (1/3 of the hexagon).

[4] This, incidentally, is at the heart of the Dynamic Pedagogy experience we hoped teachers would provide for their students.

[5] We have data from six classrooms, with 8 teachers (two classes were team taught). The 9th teacher entered the study at a later point.

[6] This and all other teacher names are pseudonyms.

[7] Students are identified with one syllable pseudonyms.

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