

**ADVANCED MATHEMATICS
CONTENT:
A COMPARATIVE ANALYSIS
OF CCSSM AND
MATHEMATICS TEXTBOOKS**

FOR TEACHERS

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OUTLINE

- **Background of Study**
- **Group Discussion – Part 1**
- **Analysis and Findings**
- **Group Discussion – Part 2**
- **Conclusions and Implications**

INTRODUCTION

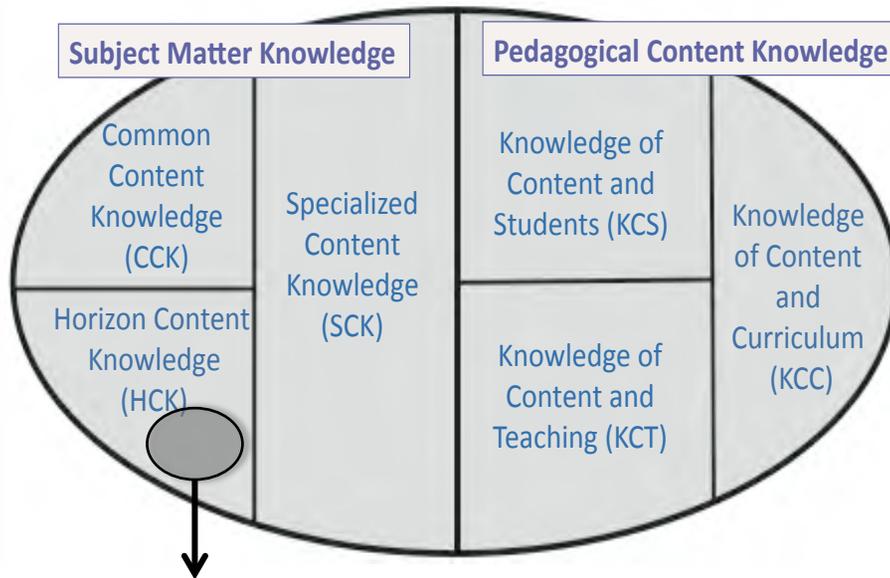
PREMISE

Effective teaching of mathematics requires various types of mathematical and pedagogical knowledge.

We look specifically at mathematical content knowledge, in particular *advanced content*.

MKT

Mathematical Knowledge for Teaching (Ball, et al.)



**Advanced-Mathematical
Horizon Knowledge (AMHK)**

Advanced-mathematics for AMHK was defined as mathematics content that was beyond what could be expected of every HS graduate (as determined by CCSSM standards)

TWO PERSPECTIVES

In this study, we compare AMHK demands from two perspectives.

- **Common Core State Standards for Mathematics**
- **Elementary and Secondary Mathematics Textbooks for Teachers**

Research Question

- **Does the *advanced-mathematical* content in these textbooks align with and support teachers' implementation of the CCSSM?**

GROUP DISCUSSION – PART 1

- **Identify advanced mathematical content (not pedagogical content) important for mathematics teachers to know.**
- **Identify advanced mathematics content currently included in your teacher education and/or professional development programs.**

CCSSM ANALYSIS

- **Two mathematics educators with teaching experience looked at every CCSSM standard and tried to answer the question:**
 - What advanced-mathematical knowledge (AMHK) would be useful for teachers in the teaching of this content?*
- **First round: Descriptions of the AMHK were written individually for each CCSSM standard**
- **Second round: Collaboration to resolve conflict and generate a final list of CCSSM standards and AMHK descriptions**
- **Coding Framework Development**
 - After multiple revisions and iterations, descriptions of 47 categories of AMHK (within 10 strands) were agreed upon by all four researchers.
- **Third round: Individual re-coding according to Coding Framework**
- **Fourth round: Validity check (~70% agreement obtained) and conflict resolution**

CODING FRAMEWORK

- One example of a description in the Coding Framework

Strand	Code	Description Teaching is enhanced by teachers' mathematical knowledge of/that...	Description As connected to (for example)...
Geometry and Measurement	2.1 Measurement Axioms	Attributes to be measured (e.g., Distance, Area, Volume, Angle, etc.) require understanding of <i>axiomatic underpinnings of measurement</i> (particularly, Countable Additivity; but also, non-negative and measure of nothing is 0) and rigorous development of a measure as a function with an appropriately defined unit	Area via Riemann sums Countable additivity as an axiom of all measurements, including how we determine length, area, volume, etc. by adding and subtracting component portions Analysis of what is a unit of measure Construction of rational numbers using Thales' Basic Proportionality Theorem
	2.2 Distance Metrics	How different definitions for distance arise from various contexts and coordinate systems (e.g., discrete metric, great circles, rotational, distance between a point and line, between two points, etc.)	The distance formula (specific to Euclidean geometry, Pythagorean Theorem) Distance on a sphere (as a plane flies) Comparing distance from a point to a line in geometry (perpendicular distance) and statistics (vertical distance)

CODING FRAMEWORK

- **NOTE: From the Coding Framework Descriptions, we realized a need to distinguish a “level” of understanding that we thought was necessary based on CCSS-M standards**
- **3 “levels” of AMHK were defined:**
 - L1) Mathematical awareness;
 - L2) Knowing the heart of an idea, working knowledge of examples & counterexamples;
 - L3) Using ideas rigorously in proof

Level 1

Level 2

Level 3

CCSSM ANALYSIS

- **Examples of CCSSM Coding:**
 - **Measure lengths indirectly and by iterating length units.**
 - **1.MD.2.** Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.*
- Coded to 2.1: Measurement Axioms

Level 2

CCSSM ANALYSIS

- **Examples of CCSSM Coding:**
 - **Draw and identify lines and angles, and classify shapes by properties of their lines and angles**
 - **4.G.2.** Classify two dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.
 - Coded to 2.6: Parallel Postulate, Non-Euclidean Geometry Level 1

CCSSM ANALYSIS

- **Examples of CCSSM Coding:**
 - **Understand the connections between proportional relationships, lines, and linear equations**
 - **F-BF.4.** Find inverse functions. Solve an equation of the form $f(x)=c$ for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x)=2x+3$ or $f(x)=(x+1)/(x-1)$ for $x \neq 1$. Verify by composition that one function is the inverse of another.
- Coded to 3.1: Group Theory Axioms

Level 3

CCSSM ANALYSIS

- **Examples of CCSSM Coding:**
 - **4.2.2: Equivalence Classes**

Grade	CCSSM	Level	Grade	CCSSM	Level
Elementary	3.NF.3	Average Level: 2.1	Middle	7.NS.2D	Average Level: 3
	3.NF.3A			8.NS.1	
	3.NF.3B				
	4.NF.1				
	4.NF.6				
	5.NF.1				
			Grade	CCSSM	Level
			High	G.CO.6	Average Level: 3
				G.CO.8	

CCSSM ANALYSIS

- **Some difficulties in coding:**
 - Distinguishing between the content covered in the standard and AMHK that could be useful for the teacher
 - AMHK descriptions are limited by the 2 mathematics educators own knowledge of mathematics, but also informed by a pedagogical perspective and their own experiences teaching
 - Distinguishing between SMK and PCK –some parts of our descriptions of AMHK may actually be primarily pedagogical (as opposed to mathematical) ideas
 - Coding to the 3 levels was, at times, difficult. In particular, distinguishing whether something fit under the 4.5 Proofs category, or another category at a level 3 was difficult.

CCSSM

Very general observations:

- In general, HS teachers need to have a proof level understanding of more ideas; whereas ES need more awareness of ideas.
- Elementary teachers do not need much (that would be considered “advanced”) in terms of Geometry (except measurement), Functions, Vectors & Matrices, and Statistics. Middle and Secondary teachers need more advanced content coverage.

Domain	AMHK	Code	Elementary	Middle	Secondary
Set Theory	One to One, Cardinality	1.1	7		1
	Set Operations and Relations	1.2	8	5	4
	Partitions and Partitioning	1.3	7		
Geometry and Measurement	Measurement Axioms	2.1	31	5	1
	Distance Metrics	2.2		6	3
	Transformations	2.3		7	11
	Analysis of Geometric Shapes in the Plane	2.4	6	2	12
	Analysis of Geometric Solids in the Plane	2.5	1	2	4
	Solids of Revolution	2.5.1		3	3
	Parallel Postulate, Non-Euclidean Geomet	2.6	1	1	6
Algebraic Structures	Group Theory Axioms	3.1	2	11	10
	Binary Operators	3.1.1	10		5
	Closure	3.1.2	2		5
	Associativity	3.1.3	8		
	Identity Element	3.1.4	2	1	1
Mathematical Foundations	Inverse Elements	3.1.5	11	6	11
	Definitions and Axioms	4.1	4	2	4
	Equivalence Relations	4.2.1	1	2	
	Equivalence Classes	4.2.2	6	2	2
	Ordering Relations	4.3.1	7	2	
	Well-Ordered Sets	4.3.2	6	1	
Number Theory	Logic	4.4		6	3
	Proof	4.5	5	3	16
	Even/Odd Numbers	5.1	2		
	Prime Numbers	5.2	1	1	1
	Divisibility Rules and Patterns	5.3	2		1
Analysis of Number Systems	Modular Arithmetic	5.4	1		
	Division Algorithm	5.5	3		2
	Base Number System	6.1.1	33	1	
	Rational Numbers	6.2	6	1	
	Integer (Negative) Numbers	6.2.1		4	
	Real (Irrational) Numbers	6.3	1	3	2
Calculus of Functions	Algebraic and Transcendental Numbers	6.4			1
	Complex Numbers	6.6			5
	Analysis of Functions	7.1		4	28
	Sequences and Series	7.2	1	1	6
	Rate of Change, Derivative	7.3		9	7
Vectors and Matrices	Polar Functions	7.4			4
	Vectors	8.1	6	1	11
Probability and Statistics	Matrices	8.2		6	11
	Univariate Statistical Concepts	9.1	1	12	6
	Bivariate Statistical Concepts	9.1.1		5	7
	Probability	9.2.1		6	13
	Mathematical Foundations of Statistics	9.2		4	8
	Combinatorics	9.3			7
Modeling	Variability	9.4	3	5	4
	Modeling and Problem Solving	10.1	3	7	29

Level 1

Level 2

Level 3

TEXTBOOK ANALYSIS

- **Identification of textbooks**
- **Initial identification of advanced content**
- **Initial alignment of identified advanced content with AMHK Framework**
- **Detailed review/alignment of textbooks with AMHK Framework**
 - Presence of content presented at a level beyond Algebra 2
 - No inclusion of levels (1-3) at this point

TEXTBOOK ANALYSIS

- **Some difficulties:**
 - Decisions whether advanced mathematics content treated in advanced way
 - Advanced content represented superficially
 - Content “looks” mathematically rigorous

TEXTBOOK ANALYSIS

			Elementary								Middle			Secondary		
			T.B. (2008A)	T.B. (2008E)	S.B. (2010)	R.B. (2010)	D.D. (2010)	G.M. (2008)	H.W. (2011)	T.S. (2010)	T.S. (2010)	E.B. (2011)	A.S. (2011)	E.B. (2011)	A.S. (2011)	Z.U. (2002)
Set Theory	One to One, Cardinality	1.1	7	7	7	7	7	7	7	7	0	0	0	1	1	1
	Set Operations and Relations	1.2	8	8	8	8	8	8	8	8	5	5	5	4	4	4
	Partitions and Partitioning	1.3	7	7	7	7	7	7	7	7	0	0	0	0	0	0
Geometry and Measurement	Measurement Axioms	2.1	31	31	31	31	31	31	31	31	5	5	5	1	1	1
	Distance Metrics	2.2	0	0	0	0	0	0	0	0	6	6	6	3	3	3
	Transformations	2.3	0	0	0	0	0	0	0	0	7	7	7	11	11	11
	Analysis of Geometric Shapes in the Plane	2.4	6	6	6	6	6	6	6	6	2	2	2	12	12	12
	Analysis of Geometric Solids in the Plane	2.5	1	1	1	1	1	1	1	1	2	2	2	4	4	4
	Solids of Revolution	2.5.1	0	0	0	0	0	0	0	0	3	3	3	3	3	3
Parallel Postulate, Non-Euclidean Geometry	2.6	1	1	1	1	1	1	1	1	1	1	1	6	6	6	
Algebraic Structures	Group Theory Axioms	3.1	2	2	2	2	2	2	2	2	11	11	11	10	10	10
	Binary Operators	3.1.1	10	10	10	10	10	10	10	10	0	0	0	5	5	5
	Closure	3.1.2	2	2	2	2	2	2	2	2	0	0	0	5	5	5
	Associativity	3.1.3	8	8	8	8	8	8	8	8	0	0	0	0	0	0
	Identity Element	3.1.4	2	2	2	2	2	2	2	2	1	1	1	1	1	1
	Inverse Elements	3.1.5	11	11	11	11	11	11	11	11	6	6	6	11	11	11
Mathematical Foundations	Definitions and Axioms	4.1	4	4	4	4	4	4	4	4	2	2	2	4	4	4
	Equivalence Relations	4.2.1	1	1	1	1	1	1	1	1	2	2	2	0	0	0
	Equivalence Classes	4.2.2	6	6	6	6	6	6	6	6	2	2	2	2	2	2
	Ordering Relations	4.3.1	7	7	7	7	7	7	7	7	2	2	2	0	0	0
	Well-Ordered Sets	4.3.2	6	6	6	6	6	6	6	6	1	1	1	0	0	0
	Logic	4.4	0	0	0	0	0	0	0	0	6	6	6	3	3	3
	Proof	4.5	5	5	5	5	5	5	5	5	3	3	3	16	16	16
Number Theory	Even/Odd Numbers	5.1	2	2	2	2	2	2	2	2	0	0	0	0	0	0
	Prime Numbers	5.2	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Divisibility Rules and Patterns	5.3	2	2	2	2	2	2	2	2	0	0	0	1	1	1
	Modular Arithmetic	5.4	1	1	1	1	1	1	1	1	0	0	0	0	0	0
	Division Algorithm	5.5	3	3	3	3	3	3	3	3	0	0	0	2	2	2
Analysis of Number Systems	Base Number System	6.1.1	33	33	33	33	33	33	33	33	1	1	1	0	0	0
	Rational Numbers	6.2	6	6	6	6	6	6	6	6	1	1	1	0	0	0
	Integer (Negative) Numbers	6.2.1	0	0	0	0	0	0	0	0	4	4	4	0	0	0
	Real (Irrational) Numbers	6.3	1	1	1	1	1	1	1	1	3	3	3	2	2	2
	Algebraic and Transcendental Numbers	6.4	0	0	0	0	0	0	0	0	0	0	0	1	1	1
	Complex Numbers	6.6	0	0	0	0	0	0	0	0	0	0	0	5	5	5
Calculus of Functions	Analysis of Functions	7.1	0	0	0	0	0	0	0	0	4	4	4	28	28	28
	Sequences and Series	7.2	1	1	1	1	1	1	1	1	1	1	1	6	6	6
	Rate of Change, Derivative	7.3	0	0	0	0	0	0	0	0	9	9	9	7	7	7
	Polar Functions	7.4	0	0	0	0	0	0	0	0	0	0	0	4	4	4
Vectors and Matrices	Vectors	8.1	6	6	6	6	6	6	6	6	1	1	1	11	11	11
	Matrices	8.2	0	0	0	0	0	0	0	0	6	6	6	11	11	11
Probability and Statistics	Univariate Statistical Concepts	9.1	1	1	1	1	1	1	1	1	12	12	12	6	6	6
	Bivariate Statistical Concepts	9.1.1	0	0	0	0	0	0	0	0	5	5	5	7	7	7
	Probability	9.2.1	0	0	0	0	0	0	0	0	6	6	6	13	13	13
	Mathematical Foundations of Statistics	9.2	0	0	0	0	0	0	0	0	4	4	4	8	8	8
	Combinatorics	9.3	0	0	0	0	0	0	0	0	0	0	0	7	7	7
	Variability	9.4	3	3	3	3	3	3	3	3	5	5	5	4	4	4
Modeling	Modeling and Problem Solving	10.1	3	3	3	3	3	3	3	3	7	7	7	29	29	29

NUMBER AND OPERATIONS

AMHK Code		Elementary	TB2008a	TB2008b	R.B. (2009)	D.D. (2010)	G.M. (2008)	T.S. (2010)	H.W. (2011)	Middle	T.S. (2010)	E.B. (2011)	A.S. (2011)	Secondary	E.B. (2011)	A.S. (2011)	Z.U. (2002)
Number Theory	5.1 Even/Odd Numbers		2										0			0	0
	5.2 Prime Numbers								1			1	1		1	1	1
	5.3 Divisibility Rules and Patterns								2			0	0		1	1	
	5.4 Modular Arithmetic			1	1		1						0			0	0
	5.5 Division Algorithm								3			0	0				
Analysis of Number Systems	6.1.1 Base Number System		33	33	33	33	33	33	33		1	1	1		0	0	0
	6.2 Rational Numbers				6							1	1		0	0	0
	6.2.1 Integer (Negative) Numbers												4		0		
	6.3 Real (Irrational) Numbers				1		1		1			3	3		2	2	2
	6.4 Algebraic & Transcendental Numbers											0	0		1	1	1
	6.6 Complex Numbers											0	0		5	5	5
Level 1		12.50%	0.00%	0.00%	100.00%	0.00%	100.00%	0.00%	100.00%	0.00%	0.00%	100.00%	100.00%	16.67%	100.00%	100.00%	100.00%
Level 2		50.00%	0.00%	25.00%	50.00%	0.00%	25.00%	0.00%	50.00%	20.00%	0.00%	100.00%	100.00%	33.33%	100.00%	100.00%	50.00%
Level 3		37.50%	66.67%	33.33%	33.33%	33.33%	33.33%	33.33%	66.67%	80.00%	25.00%	75.00%	100.00%	50.00%	100.00%	100.00%	100.00%
Total Levels		100.00%	25.00%	25.00%	50.00%	12.50%	37.50%	12.50%	62.50%	100.00%	20.00%	80.00%	100.00%	100.00%	100.00%	100.00%	83.33%

AMHK and CCSSM

- Rows show AMHK categories and codes
- Colors refer to AMHK levels
- Numbers in cells refer to the number of standards to which the AMHK code applies.

AMHK and Textbooks

- Columns are grade bands (yellow) and textbooks.
- Cells with border and number indicate inclusion of the AMHK code in the textbook
- Level and number only apply to CCSSM

NUMBER AND OPERATIONS

AMHK Code		Elementary	TB2008a	TB2008b	R.B. (2009)	D.D. (2010)	G.M. (2008)	T.S. (2010)	H.W. (2011)	Middle	T.S. (2010)	E.B. (2011)	A.S. (2011)	Secondary	E.B. (2011)	A.S. (2011)	Z.U. (2002)
Number Theory	5.1 Even/Odd Numbers		2										0			0	0
	5.2 Prime Numbers								1			1	1		1	1	1
	5.3 Divisibility Rules and Patterns								2			0	0		1	1	
	5.4 Modular Arithmetic			1	1		1						0			0	0
	5.5 Division Algorithm								3			0	0		2	2	2
Analysis of Number Systems	6.1.1 Base Number System		33	33	33	33	33	33	33		1	1	1		0	0	0
	6.2 Rational Numbers				6							1	1		0	0	0
	6.2.1 Integer (Negative) Numbers												4			0	
	6.3 Real (Irrational) Numbers				1		1		1			3	3		2	2	2
	6.4 Algebraic & Transcendental Numbers											0	0		1	1	1
	6.6 Complex Numbers											0	0		5	5	5
Level 1		12.50%	0.00%	0.00%	100.00%	0.00%	100.00%	0.00%	100.00%	0.00%	0.00%	100.00%	100.00%	16.67%	100.00%	100.00%	100.00%
Level 2		50.00%	0.00%	25.00%	50.00%	0.00%	25.00%	0.00%	50.00%	20.00%	0.00%	100.00%	100.00%	33.33%	100.00%	100.00%	50.00%
Level 3		37.50%	66.67%	33.33%	33.33%	33.33%	33.33%	33.33%	66.67%	80.00%	25.00%	75.00%	100.00%	50.00%	100.00%	100.00%	100.00%
Total Levels		100.00%	25.00%	25.00%	50.00%	12.50%	37.50%	12.50%	62.50%	100.00%	20.00%	80.00%	100.00%	100.00%	100.00%	100.00%	83.33%

AMHK by Level and Grade Band

Yellow - Grade Band

- Refers only to the AMHK codes included in the CCSSM grade band
- Levels indicate the percent of included AMHK at each level for grade band

Textbooks

- Percent of included codes in the CCSSM grade band present in the textbook by level and total
- Number of included codes vary by level and grade band.

NUMBER AND OPERATIONS

AMHK Code		Elementary							Middle			Secondary			
		TB2008a	TB2008b	R.B. (2009)	D.D. (2010)	G.M. (2008)	T.S. (2010)	H.W. (2011)	T.S. (2010)	E.B. (2011)	A.S. (2011)	E.B. (2011)	A.S. (2011)	Z.U. (2002)	
Number Theory	5.1 Even/Odd Numbers	2									0		0	0	
	5.2 Prime Numbers							1	1		1	1	1	1	
	5.3 Divisibility Rules and Patterns								0	0		1	1		
	5.4 Modular Arithmetic		1	1		1							0	0	
	5.5 Division Algorithm							3				2	2	2	
Analysis of Number Systems	6.1.1 Base Number System	33	33	33	33	33	33	33	1	1	1	0	0	0	
	6.2 Rational Numbers			6						1	1	0	0	0	
	6.2.1 Integer (Negative) Numbers										4		0		
	6.3 Real (Irrational) Numbers			1		1		1				2	2	2	
	6.4 Algebraic & Transcendental Numbers								3	3		1	1	1	
	6.6 Complex Numbers								0	0		5	5	5	
Level 1		12.50%	0.00%	0.00%	100.00%	0.00%	100.00%	0.00%	100.00%	0.00%		16.67%	100.00%	100.00%	100.00%
Level 2		50.00%	0.00%	25.00%	50.00%	0.00%	25.00%	0.00%	50.00%	20.00%	0.00%	100.00%	100.00%	50.00%	
Level 3		37.50%	66.67%	33.33%	33.33%	33.33%	33.33%	33.33%	66.67%	80.00%	25.00%	75.00%	100.00%	100.00%	
Total Levels		100.00%	25.00%	25.00%	50.00%	12.50%	37.50%	12.50%	62.50%	100.00%	20.00%	80.00%	100.00%	83.33%	

Elementary (1,4,3)

- CCSSM emphasis on Number Theory,
- Sparse inclusion of AMHK codes in textbooks,
- Only *Base Number Systems* included in all textbooks.

Middle (0,1,4)

- CCSSM emphasis on *Number Systems*,
- CCSSM emphasis on level 3 AMHK codes,
- MS/HS greater inclusion than ES/MS textbooks.

Secondary (1,2,3)

- 3 of 5 *Number Theory* and 3 of 6 *Number System* AMHK codes,
- High inclusion of AMHK codes in textbooks,
- High consistency across textbooks.

GEOMETRY AND MEASUREMENT

	AMHK Code	Elementary	Middle						Middle	Secondary			Secondary	Secondary		
			T.B. (2008A)	T.B. (2008B)	R.B. (2009)	D.D. (2010)	G.M. (2008)	T.S. (2010)		H.W. (2011)	T.S. (2010)	E.B. (2011)		A.S. (2011)	E.B. (2011)	A.S. (2011)
Geometry and Measurement	2.1 Measurement Axioms											5			1	1
	2.2 Distance Metrics															3
	2.3 Transformations		0									7			11	11
	2.4 Analysis of Geometric Shapes in the Plane				6							2	2	12	12	12
	2.5 Analysis of Geometric Solids in Space											2			4	4
	2.5.1 Solids of Revolution											3			3	
	2.6 Parallel Postulate, Non-Euclidean Geometry											1			6	6
Level 1	50.00%	0.00%	0.00%	0.00%	50.00%	0.00%	0.00%	0.00%	14.29%	0.00%	0.00%	100.00%	0.00%			
Level 2	50.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	85.71%	0.00%	16.67%	83.33%	71.43%	0.00%	80.00%	100.00%
Level 3	0.00%								0.00%				28.57%	50.00%	100.00%	50.00%
Total Levels	100.00%	0.00%	0.00%	0.00%	25.00%	0.00%	0.00%	0.00%	100.00%	0.00%	14.29%	85.71%	100.00%	14.29%	85.71%	85.71%

Elementary

- CCSSM level 1 and 2 codes,
- Virtually no inclusion of AMHK codes in textbooks,
- Notable absence of advanced measurement content.

Middle

- CCSSM emphasis on level 2 AMHK codes,
- ES/MS text no inclusion of codes,

- MS/HS difference in inclusion of codes.

Secondary

- CCSSM level 2 and 3 codes

ALGEBRA

	AMHK Code	Elementary	T.B. (2008A)	T.B. (2008B)	S.B. (2010)	R.B. (2009)	D.D. (2010)	G.M. (2008)	T.S. (2010)	H.W. (2011)	Middle	T.S. (2010)	E.B. (2011)	A.S. (2011)	Secondary	E.B. (2011)	A.S. (2011)	Z.U. (2002)
Algebraic Structures	3.1 Group Theory Axioms																	10
	3.1.1 Binary Operators																	5
	3.1.2 Closure																	5
	3.1.3 Associativity																	0
	3.1.4 Identity Element																	1
	3.1.5 Inverse Element																	11
Calculus of Functions	7.1 Analysis of Functions												4	4		28	28	28
	7.2 Sequences and Series			1														6
	7.3 Rate of Change, Derivative																	7
	7.4 Polar Functions																	
Vectors and Matrices	8.1 Vectors									6			1	1		11	11	
	8.2 Matrices													6			11	11
	Level 1	25.00%	0.00%	0.00%	50.00%	0.00%	0.00%	0.00%	0.00%	0.00%	12.50%	0.00%	0.00%	0.00%	0.00%			
	Level 2	62.50%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	20.00%	50.00%	0.00%	25.00%	50.00%	18.18%	0.00%	0.00%	50.00%
	Level 3	12.50%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	37.50%	0.00%	33.33%	33.33%	81.82%	22.22%	33.33%	88.89%
	Total Levels	100.00%	0.00%	0.00%	12.50%	0.00%	0.00%	0.00%	0.00%	12.50%	100.00%	0.00%	25.00%	37.50%	100.00%	18.18%	27.27%	81.82%

- Throughout K-12 CCSSM, standards had links to algebraic structures (e.g., groups); None of the elementary or middle textbooks, and only one secondary textbook contained an advanced treatment of the subject.

ALGEBRA

ALGEBRAIC STRUCTURES, by CCSS-M grade level

Strand	AMHK	Code	K	1	2	3	4	5	6	7	8	HS
Algebraic Structures	Group Theory Axioms	3.1				1		1	3	7	1	1
	Binary Operators	3.1.1	1	1	3	4	1					5
	Closure	3.1.2				1		1				5
	Associativity	3.1.3	1	2	1	1	1	2				
	Identity Element	3.1.4				1	1		1			1
	Inverse Elements	3.1.5		3	3	3		2	3	3		11

- While arithmetic properties were frequently covered in textbooks, (e.g. -2 is the additive inverse of 2) there was little rigor in the treatment (e.g. existence and uniqueness of inverses, or the connection to an identity element) beyond what they would know from their own schooling
- For secondary, is lack of coverage an expectation that students would have already seen that in their math major courses?

ALGEBRA

	AMHK Code	Elementary	T.B. (2008A)	T.B. (2008B)	S.B. (2010)	R.B. (2009)	D.D. (2010)	G.M. (2008)	T.S. (2010)	H.W. (2011)	Middle	T.S. (2010)	E.B. (2011)	A.S. (2011)	Secondary	E.B. (2011)	A.S. (2011)	Z.U. (2002)
Algebraic Structures	3.1 Group Theory Axioms																	10
	3.1.1 Binary Operators																	5
	3.1.2 Closure																	5
	3.1.3 Associativity																	0
	3.1.4 Identity Element																	1
	3.1.5 Inverse Element																	11
Calculus of Functions	7.1 Analysis of Functions											4	4			28	28	28
	7.2 Sequences and Series			1														6
	7.3 Rate of Change, Derivative																	7
	7.4 Polar Functions																	
Vectors and Matrices	8.1 Vectors									6			1	1		11	11	
	8.2 Matrices													6			11	11
	Level 1	25.00%	0.00%	0.00%	50.00%	0.00%	0.00%	0.00%	0.00%	0.00%	12.50%	0.00%	0.00%	0.00%	0.00%			
	Level 2	62.50%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	20.00%	50.00%	0.00%	25.00%	50.00%	18.18%	0.00%	0.00%	50.00%
	Level 3	12.50%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	37.50%	0.00%	33.33%	33.33%	81.82%	22.22%	33.33%	88.89%
	Total Levels	100.00%	0.00%	0.00%	12.50%	0.00%	0.00%	0.00%	0.00%	12.50%	100.00%	0.00%	25.00%	37.50%	100.00%	18.18%	27.27%	81.82%

- While many Middle & Secondary textbooks covered Analysis of Functions, the other topics were left uncovered
- In particular, the lack of connection to derivatives in Middle

ALGEBRA

	AMHK Code	Elementary	T.B. (2008A)	T.B. (2008B)	S.B. (2010)	R.B. (2009)	D.D. (2010)	G.M. (2008)	T.S. (2010)	H.W. (2011)	Middle	T.S. (2010)	E.B. (2011)	A.S. (2011)	Secondary	E.B. (2011)	A.S. (2011)	Z.U. (2002)
Algebraic Structures	3.1 Group Theory Axioms																	10
	3.1.1 Binary Operators																	5
	3.1.2 Closure																	5
	3.1.3 Associativity																	0
	3.1.4 Identity Element																	1
	3.1.5 Inverse Element																	11
Calculus of Functions	7.1 Analysis of Functions												4	4		28	28	28
	7.2 Sequences and Series			1														6
	7.3 Rate of Change, Derivative																	7
	7.4 Polar Functions																	
Vectors and Matrices	8.1 Vectors									6			1	1		11	11	
	8.2 Matrices												6				11	11
	Level 1	25.00%	0.00%	0.00%	50.00%	0.00%	0.00%	0.00%	0.00%	0.00%	12.50%	0.00%	0.00%	0.00%	0.00%			
	Level 2	62.50%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	20.00%	50.00%	0.00%	25.00%	50.00%	18.18%	0.00%	0.00%	50.00%
	Level 3	12.50%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	37.50%	0.00%	33.33%	33.33%	81.82%	22.22%	33.33%	88.89%
	Total Levels	100.00%	0.00%	0.00%	12.50%	0.00%	0.00%	0.00%	0.00%	12.50%	100.00%	0.00%	25.00%	37.50%	100.00%	18.18%	27.27%	81.82%

- Despite the pedagogical potential of vectors in elementary & middle, content is in few textbooks.
- Many secondary CCSS-M (+) explicitly cover Vectors and Matrices; others utilize knowledge applied from such knowledge to things like the complex plane and transformations.
- Transformational approach advocated by CCSS-M makes transformational matrices important.

DATA ANALYSIS AND PROBABILITY

	AMHK Code	Elementary	T.B. (2008A)	T.B. (2008B)	S.B. (2010)	R.B. (2009)	D.D. (2010)	G.M. (2008)	T.S. (2010)	H.W. (2011)	Middle	T.S. (2010)	E.B. (2011)	A.S. (2011)	Secondary	E.B. (2011)	A.S. (2011)	Z.U. (2002)
Probability and Statistics	9.1 Univariate Statistical Concepts		1		1	1		1	1			12		12			6	
	9.1.1 Bivariate Statistical Concepts													5			7	
	9.2.1 Probability				0	0		0						6			13	
	9.2 Mathematical Foundations of				0	0												
	9.3 Combinatorics													0			7	
	9.4 Variability																	
	Level 1	100.00%	50.00%	0.00%	50.00%	50.00%	0.00%	50.00%	50.00%	0.00%	20.00%	0.00%	0.00%	0.00%	0.00%			
	Level 2	0.00%									80.00%	25.00%	0.00%	75.00%	83.33%	0.00%	60.00%	0.00%
	Level 3	0.00%									0.00%			16.67%	0.00%	100.00%	0.00%	
	Total Levels	100.00%	50.00%	0.00%	50.00%	50.00%	0.00%	50.00%	50.00%	0.00%	100.00%	20.00%	0.00%	60.00%	100.00%	0.00%	66.67%	0.00%

- **Introductory (Level 1) knowledge of statistical concepts for teachers begins in Elementary grades, with measurement concepts.**
- **Even in Secondary grades, teachers likely do not need to be able to have proof level knowledge (e.g., understanding conceptually the idea of least squares regression line, versus being able to prove the equation of the line from data.)**
- **Overall, textbooks covered some statistical ideas, but sporadically.**

GROUP DISCUSSION – PART 2

Describe at least two findings that were interesting or surprising to you.

How could the findings inform your teacher education/professional development programs?

SOME GENERAL THEMES

Among the elementary and elementary/middle textbooks, there was very little consistency in advanced content. Except Base Number Systems, which was covered by nearly all textbooks, any other advanced content that was covered was seen in less than 50% of the textbooks.

		Elementary										Middle			
Set Theory	One to One, Cardinality	1.1	7	7	7	7	7	7	7	7	7	7	0	0	0
	Set Operations and Relations	1.2	8	8	8	8	8	8	8	8	8	8	5	5	5
	Partitions and Partitioning	1.3	7	7	7	7	7	7	7	7	7	7	0	0	0
Geometry and Measurement	Measurement Axioms	2.1	31	31	31	31	31	31	31	31	31	31	5	5	5
	Distance Metrics	2.2	0	0	0	0	0	0	0	0	0	0	6	6	6
	Transformations	2.3	0	0	0	0	0	0	0	0	0	0	7	7	7
	Analysis of Geometric Shapes in the Plane	2.4	6	6	6	6	6	6	6	6	6	6	2	2	2
	Analysis of Geometric Solids in the Plane	2.5	1	1	1	1	1	1	1	1	1	1	2	2	2
	Solids of Revolution	2.5.1	0	0	0	0	0	0	0	0	0	0	3	3	3
Parallel Postulate, Non-Euclidean Geom.	2.6	1	1	1	1	1	1	1	1	1	1	1	1	1	
Algebraic Structures	Group Theory Axioms	3.1	2	2	2	2	2	2	2	2	2	2	11	11	11
	Binary Operators	3.1.1	10	10	10	10	10	10	10	10	10	10	0	0	0
	Closure	3.1.2	2	2	2	2	2	2	2	2	2	2	0	0	0
	Associativity	3.1.3	8	8	8	8	8	8	8	8	8	8	0	0	0
	Identity Element	3.1.4	2	2	2	2	2	2	2	2	2	2	1	1	1
	Inverse Elements	3.1.5	11	11	11	11	11	11	11	11	11	11	6	6	6
Mathematical Foundations	Definitions and Axioms	4.1	4	4	4	4	4	4	4	4	4	4	2	2	2
	Equivalence Relations	4.2.1	1	1	1	1	1	1	1	1	1	1	2	2	2
	Equivalence Classes	4.2.2	6	6	6	6	6	6	6	6	6	6	2	2	2
	Ordering Relations	4.3.1	7	7	7	7	7	7	7	7	7	7	2	2	2
	Well-Ordered Sets	4.3.2	6	6	6	6	6	6	6	6	6	6	1	1	1
	Logic	4.4	0	0	0	0	0	0	0	0	0	0	6	6	6
Proof	4.5	5	5	5	5	5	5	5	5	5	5	3	3	3	
Number Theory	Even/Odd Numbers	5.1	2	2	2	2	2	2	2	2	2	2	0	0	0
	Prime Numbers	5.2	1	1	1	1	1	1	1	1	1	1	1	1	1
	Divisibility Rules and Patterns	5.3	2	2	2	2	2	2	2	2	2	2	0	0	0
	Modular Arithmetic	5.4	1	1	1	1	1	1	1	1	1	1	0	0	0
	Division Algorithm	5.5	3	3	3	3	3	3	3	3	3	3	0	0	0
	Base Number Systems	6.1.1	33	33	33	33	33	33	33	33	33	33	1	1	1
Analysis of Number Systems	Rational Numbers	6.2	6	6	6	6	6	6	6	6	6	6	1	1	1
	Integer (Negative) Numbers	6.2.1	0	0	0	0	0	0	0	0	0	0	4	4	4
	Real (Irrational) Numbers	6.3	1	1	1	1	1	1	1	1	1	1	3	3	3
	Algebraic and Transcendental Numbers	6.4	0	0	0	0	0	0	0	0	0	0	0	0	0
	Complex Numbers	6.6	0	0	0	0	0	0	0	0	0	0	0	0	0
	Calculus of Functions	Analysis of Functions	7.1	0	0	0	0	0	0	0	0	0	0	4	4
Sequences and Series		7.2	1	1	1	1	1	1	1	1	1	1	1	1	1
Rate of Change, Derivative		7.3	0	0	0	0	0	0	0	0	0	0	9	9	9
Polar Functions		7.4	0	0	0	0	0	0	0	0	0	0	0	0	0
Vectors and Matrices	Vectors	8.1	6	6	6	6	6	6	6	6	6	6	1	1	1
	Matrices	8.2	0	0	0	0	0	0	0	0	0	0	6	6	6
Probability and Statistics	Univariate Statistical Concepts	9.1	1	1	1	1	1	1	1	1	1	1	12	12	12
	Bivariate Statistical Concepts	9.1.1	0	0	0	0	0	0	0	0	0	0	5	5	5
	Probability	9.2.1	0	0	0	0	0	0	0	0	0	0	6	6	6
	Mathematical Foundations of Statistics	9.2	0	0	0	0	0	0	0	0	0	0	4	4	4
	Combinatorics	9.3	0	0	0	0	0	0	0	0	0	0	0	0	0
Variability	9.4	3	3	3	3	3	3	3	3	3	3	5	5	5	
Modeling	Modeling and Problem Solving	10.1	3	3	3	3	3	3	3	3	3	7	7	7	

SOME GENERAL THEMES

In addition, large aspects of advanced content (that teachers should know at a high level) that could be useful for teaching the CCSSM standard were missing completely.

Also, for the many aspects the textbooks did cover, they were not necessarily needed at a particularly deep level.

			Elementary								Middle			
Set Theory	One to One, Cardinality	1.1	7	7	7	7	7	7	7	7	7	0	0	0
	Set Operations and Relations	1.2	8	8	8	8	8	8	8	8	8	5	5	5
	Partitions and Partitioning	1.3	7	7	7	7	7	7	7	7	7	0	0	0
Geometry and Measurement	Measurement Axioms	2.1	31	31	31	31	31	31	31	31	31	5	5	5
	Distance Metrics	2.2	0	0	0	0	0	0	0	0	0	6	6	6
	Transformations	2.3	0	0	0	0	0	0	0	0	0	7	7	7
	Analysis of Geometric Shapes in the Plane	2.4	6	6	6	6	6	6	6	6	6	2	2	2
	Analysis of Geometric Solids in the Plane	2.5	1	1	1	1	1	1	1	1	1	2	2	2
	Solids of Revolution	2.5.1	0	0	0	0	0	0	0	0	0	0	3	3
Algebraic Structures	Parallel Postulate, Non-Euclidean Geom.	2.6	1	1	1	1	1	1	1	1	1	1	1	1
	Group Theory Axioms	3.1	2	2	2	2	2	2	2	2	2	11	11	11
	Binary Operators	3.1.1	10	10	10	10	10	10	10	10	10	0	0	0
	Closure	3.1.2	2	2	2	2	2	2	2	2	2	0	0	0
	Associativity	3.1.3	8	8	8	8	8	8	8	8	8	0	0	0
	Identity Element	3.1.4	2	2	2	2	2	2	2	2	2	1	1	1
Mathematical Foundations	Inverse Elements	3.1.5	11	11	11	11	11	11	11	11	11	6	6	6
	Definitions and Axioms	4.1	4	4	4	4	4	4	4	4	4	2	2	2
	Equivalence Relations	4.2.1	1	1	1	1	1	1	1	1	1	2	2	2
	Equivalence Classes	4.2.2	6	6	6	6	6	6	6	6	6	2	2	2
	Ordering Relations	4.3.1	7	7	7	7	7	7	7	7	7	2	2	2
	Well-Ordered Sets	4.3.2	6	6	6	6	6	6	6	6	6	1	1	1
Number Theory	Logic	4.4	0	0	0	0	0	0	0	0	0	6	6	6
	Proof	4.5	5	5	5	5	5	5	5	5	5	3	3	3
	Even/Odd Numbers	5.1	2	2	2	2	2	2	2	2	2	0	0	0
	Prime Numbers	5.2	1	1	1	1	1	1	1	1	1	1	1	1
	Divisibility Rules and Patterns	5.3	2	2	2	2	2	2	2	2	2	0	0	0
	Modular Arithmetic	5.4	1	1	1	1	1	1	1	1	1	0	0	0
Analysis of Number Systems	Division Algorithm	5.5	3	3	3	3	3	3	3	3	3	0	0	0
	Base Number System	6.1.1	33	33	33	33	33	33	33	33	33	1	1	1
	Rational Numbers	6.2	6	6	6	6	6	6	6	6	6	1	1	1
	Integer (Negative) Numbers	6.2.1	0	0	0	0	0	0	0	0	0	4	4	4
	Real (Irrational) Numbers	6.3	1	1	1	1	1	1	1	1	1	3	3	3
	Algebraic and Transcendental Numbers	6.4	0	0	0	0	0	0	0	0	0	0	0	0
Calculus of Functions	Complex Numbers	6.6	0	0	0	0	0	0	0	0	0	0	0	0
	Analysis of Functions	7.1	0	0	0	0	0	0	0	0	0	4	4	4
	Sequences and Series	7.2	1	1	1	1	1	1	1	1	1	1	1	1
	Rate of Change, Derivative	7.3	0	0	0	0	0	0	0	0	0	9	9	9
Vectors and Matrices	Polar Functions	7.4	0	0	0	0	0	0	0	0	0	0	0	0
	Vectors	8.1	6	6	6	6	6	6	6	6	6	1	1	1
Probability and Statistics	Matrices	8.2	0	0	0	0	0	0	0	0	0	6	6	6
	Univariate Statistical Concepts	9.1	1	1	1	1	1	1	1	1	1	12	12	12
	Bivariate Statistical Concepts	9.1.1	0	0	0	0	0	0	0	0	0	5	5	5
	Probability	9.2.1	0	0	0	0	0	0	0	0	0	6	6	6
	Mathematical Foundations of Statistics	9.2	0	0	0	0	0	0	0	0	0	4	4	4
Modeling	Combinatorics	9.3	0	0	0	0	0	0	0	0	0	0	0	0
	Variability	9.4	3	3	3	3	3	3	3	3	3	5	5	5
	Modeling and Problem Solving	10.1	3	3	3	3	3	3	3	3	3	7	7	7

SOME GENERAL THEMES

The 3 secondary textbooks seemed to have particular biases on advanced content.

- E.B. (2011) had very little Statistics content, Algebraic Structures, or Geometry & Measurement.
- A.S. (2011) had very little Mathematical Foundations, or Algebraic structures
- Z.U. (2002) did not cover Statistics content.
 - This may be due to the date of publication

			Secondary		
Set Theory	One to One, Cardinality	1.1	1	1	1
	Set Operations and Relations	1.2	4	4	4
	Partitions and Partitioning	1.3	0	0	0
Geometry and Measurement	Measurement Axioms	2.1	1	1	1
	Distance Metrics	2.2	3	3	3
	Transformations	2.3	11	11	11
	Analysis of Geometric Shapes in the Plane	2.4	12	12	12
	Analysis of Geometric Solids in the Plane	2.5	4	4	4
	Solids of Revolution	2.5.1	3	3	3
Algebraic Structures	Parallel Postulate, Non-Euclidean Geoms	2.6	6	6	6
	Group Theory Axioms	3.1	10	10	10
	Binary Operators	3.1.1	5	5	5
	Closure	3.1.2	5	5	5
	Associativity	3.1.3	0	0	0
Mathematical Foundations	Identity Element	3.1.4	1	1	1
	Inverse Elements	3.1.5	11	11	11
	Definitions and Axioms	4.1	4	4	4
	Equivalence Relations	4.2.1	0	0	0
	Equivalence Classes	4.2.2	2	2	2
	Ordering Relations	4.3.1	0	0	0
Number Theory	Well-Ordered Sets	4.3.2	0	0	0
	Logic	4.4	3	3	3
	Proof	4.5	16	16	16
	Even/Odd Numbers	5.1	0	0	0
	Prime Numbers	5.2	1	1	1
Analysis of Number Systems	Divisibility Rules and Patterns	5.3	1	1	1
	Modular Arithmetic	5.4	0	0	0
	Division Algorithm	5.5	2	2	2
	Base Number System	6.1.1	0	0	0
	Rational Numbers	6.2	0	0	0
Calculus of Functions	Integer (Negative) Numbers	6.2.1	0	0	0
	Real (Irrational) Numbers	6.3	2	2	2
	Algebraic and Transcendental Numbers	6.4	1	1	1
	Complex Numbers	6.6	5	5	5
	Analysis of Functions	7.1	28	28	28
Vectors and Matrices	Sequences and Series	7.2	6	6	6
	Rate of Change, Derivative	7.3	7	7	7
	Polar Functions	7.4	4	4	4
	Vectors	8.1	11	11	11
Probability and Statistics	Matrices	8.2	11	11	11
	Univariate Statistical Concepts	9.1	6	6	6
	Bivariate Statistical Concepts	9.1.1	7	7	7
	Probability	9.2.1	13	13	13
	Mathematical Foundations of Statistics	9.2	8	8	8
Modeling	Combinatorics	9.3	7	7	7
	Variability	9.4	4	4	4
	Modeling and Problem Solving	10.1	29	29	29

IMPLICATIONS

- **Design of pre-service preparation and in-service professional development**
 - Framework for mathematics textbook and course development
 - Change teaching of content related to advanced ideas
 - Expand how teachers conceptualize K-12 content
 - Improve student achievement