Siegler training: Benefits (Or Occupational Hazards?)

1. See overlapping waves everywhere
2. Require a sound experimental design always
3. Never discount individual differences
4. Central importance of children’s thinking

Translating Knowledge of Children’s Thinking to Improve Education

Julie L. Booth, Ph.D.
Temple University
Why do we need to improve math education (in the United States)?

- Consistent underperformance on state tests of mathematical proficiency (e.g., Kim, Schneider, Engec, & Siskind, 2006; Pennsylvania Department of Education, 2011)
- Poor showing in international comparisons (TIMSS; PISA)
- Aspects of early mathematics instruction can interfere with later learning (McNeil et al., 2006)
- Misconceptions abound (e.g., Booth, Barbieri, Eyer, & Pare-Blagoev, 2014)
- Students are not adequately prepared to tackle difficult gatekeepers such as fractions (Booth & Newton, 2012) and algebra (Department of Education, 1997),

How can knowledge of children’s thinking/learning be used to improve Math Education?

- Decades of work in fields of Cognitive Science/Cognitive Development
  - 10 years of NSF Science of Learning Centers
  - Compilations: Pashler et al., 2007; Dunlosky, Rawson et al., 2013; Koedinger, Booth & Klahr, 2013
- Critical gap between research and practice (NRC, 2003)
- Two proposed venues for cognitive science to make a difference in Math Education
  - Instructional Materials/Textbooks
  - Teacher Preparation
Goal: Use cognitive science principles to enhance student learning in Algebra
• Self-Explaining Correct and Incorrect Worked Examples

Self-Explanation Principle (e.g., Chi et al., 1994)
• Explaining information to yourself as you read or study
  • Facilitates integration of new information with prior knowledge
  • Forces learner to make their knowledge explicit
  • Prompts learner to generate inferences to fill gaps in their knowledge

• Also applicable in problem solving: How did you get your answer?
Worked Example Principle
(e.g., Sweller, 1999)

- Replacing some problems in a practice session with an example of how to solve a problem
- Reduces working memory load (compared with long strings of practice problems), so learners can focus on learning the steps in problem solving
- Allows students to process the information more deeply when not just routinely applying procedures
  - May naturally generate more self-explanations

Incorrect Worked Examples
(e.g., Siegler, 2002)

- Showing students common incorrect ways to solve problems, and have learner explain why the procedure is inappropriate
- Provides negative feedback, which reduces the relative strength of incorrect strategies
  - Helps them accept that the procedure is wrong
- Forces students to see the differences between the presented problem and others where a procedure does work
  - Fixes misconceptions
AlgebraByExample consists of worksheets that prompt students to study and explain correct and incorrect examples during problem solving practice.

Each example targets a critical concept or misconception for that topic area.

Does AlgebraByExample improve student learning in Algebra for diverse populations in real-world classrooms?
Participants

- 25 non-Honors Algebra 1 classrooms in 5 MSAN districts (13 Experimental, 12 Control)
- 11 Teachers
- 380 students
  - 47% male
  - 51% low SES
  - 30% White, 39% Black, 18% Hispanic, 7% Asian, and 6% biracial (63% URM)
Prior Knowledge

- **81 Items**
- What Algebra teachers would like their students to know before Algebra

Evaluate each expression for the values $x = 2, y = -3$, and $z = -4$.

- a) $5x + y - z$
- b) $x - 3(y + z)$
- c) $\frac{x}{2} + 2y$

State whether each of the following is equivalent to $x + 4 - 2 + x$:

- a. $(x + 4) - (2 + x)$ Yes No
- b. $4 + x - 2 + x$ Yes No
- c. $x + (4 - 2) + x$ Yes No
- d. $x + 4 - x + 2$ Yes No
- e. $(x + 4) + (-2 + x)$ Yes No
- f. $x + 4 + x - 2$ Yes No
- g. $x + 2(2 - 1) + x$ Yes No

Conceptual Posttest

- **41 Items**
- Understanding of the features in the content and how they relate to each other

State how many terms the resulting expression will have:

- a. $2x(3 + 4x)$ 2 3 4
- b. $(2x + 3)(2x - 1)$ 2 3 4
- c. $(x + 2)^2$ 2 3 4
- d. $(x + 4)(x - 4)$ 2 3 4
- e. $2(3x^2 - 4x + 1)$ 2 3 4
- f. $(2x + 3)(4x - 1)$ 2 3 4

State whether each of the following is true for the quadratic function $y = -x^2 - 2x + 3$:

- a. The axis of symmetry is $x = 1$. Yes No
- b. The vertex is a minimum. Yes No
- c. The vertex is $(1,4)$. Yes No
- d. The vertex is $(1,0)$. Yes No
- e. The vertex is $(-1,4)$. Yes No
- f. The vertex is $(-1,0)$. Yes No
- g. The vertex is $(4,4)$. Yes No
- h. The vertex is $(4,0)$. Yes No
Procedural Posttest

- 25 Items
- Carrying out procedures to solve problems

Simplify each expression using only positive exponents.

a. \(a^2b^1c^1\)

b. \((m^3k^2)^3\)

Factor each expression completely.

a. \(3x^2 - 18x\)

Standardized Test Items

- 10 items
- Released items from standardized tests used by participating districts

Jill is solving the equation \(7n - 6 = 15\). The result of her first step is \(7n = 21\). What operation did Jill use in her first step?

a. add 6 to each side
b. subtract 6 from each side
c. multiply both sides by 6
d. divide both sides by 6

Which of the following equations represents the relationship between \(x\) and \(y\) in the table?

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
</tr>
<tr>
<td>4</td>
<td>14</td>
</tr>
</tbody>
</table>

a. \(y = 2x\)
b. \(y = x + 2\)
c. \(y = 5x\)
d. \(y = 3x + 2\)
Design

- Pretest: Prior knowledge
- Throughout the year: AlgebraByExample assignments when deemed appropriate by teachers
- Posttest: Conceptual and procedural tests, plus released items from standardized tests; teacher report of frequency of in-class review
Results: Conceptual Posttest

Predictors of Posttest Conceptual Scores

<table>
<thead>
<tr>
<th></th>
<th>Model 1</th>
<th>Model 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fixed Effects Model</td>
<td>Interaction Model</td>
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<tr>
<td></td>
<td>Student-Level</td>
<td>Classroom-Level</td>
</tr>
<tr>
<td>URM</td>
<td>-.01 (.02)</td>
<td>.01 (.02)</td>
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<tr>
<td>Intercept</td>
<td>.34*** (.05)</td>
<td>.33*** (.05)</td>
</tr>
<tr>
<td>Treatment</td>
<td>.06 (2)</td>
<td>.06 (2)</td>
</tr>
<tr>
<td>Assignments used</td>
<td>.02 (.02)</td>
<td>.02 (.02)</td>
</tr>
<tr>
<td>Frequency of review</td>
<td>.04 (.02)</td>
<td>.04 (.02)</td>
</tr>
<tr>
<td>Treatment x Prior Knowledge interaction</td>
<td>--</td>
<td>-.34** (.12)</td>
</tr>
<tr>
<td>Treatment x URM interaction</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Random Effects</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student-Level</td>
<td></td>
</tr>
<tr>
<td>Prior Knowledge Score</td>
<td>.21 (.08)</td>
</tr>
<tr>
<td>Pretest Interest Score</td>
<td>.03 (.03)</td>
</tr>
<tr>
<td>Pretest Competence Expectancy Score</td>
<td>-.05 (.07)</td>
</tr>
<tr>
<td>Assignment Effort—Right-Hand</td>
<td>.01 (.02)</td>
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<tr>
<td>Assignment Effort—Left-Hand</td>
<td>.05 (.05)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Classroom-Level</th>
<th>Student-Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportion Reduction in Variance (from Model 1)</td>
<td>.0191</td>
<td>.0191</td>
</tr>
</tbody>
</table>

Notes. URM = Underrepresented minority; *p < .05; **p < .01; --not included in model.


Results: AlgebraByExample improves Conceptual knowledge

Gains in conceptual knowledge with AlgebraByExample even greater for struggling students (up to 10 percentage points)!

![Graph showing gains in conceptual knowledge]


Results: Procedural Posttest

<table>
<thead>
<tr>
<th>Predictors of Posttest Procedural Scores</th>
<th>Model 1 - Main Effects Model</th>
<th>Model 2 - Interaction Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed Effects</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Student-Level</td>
<td></td>
<td></td>
</tr>
<tr>
<td>URM</td>
<td>-.01 (.02)</td>
<td>.01 (.03)</td>
</tr>
<tr>
<td>Classroom-Level</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>.11* (.05)</td>
<td>.10* (.05)</td>
</tr>
<tr>
<td>Treatment (control composition)</td>
<td>.01 (.01)</td>
<td>.08 (.03)</td>
</tr>
<tr>
<td>Assignments used</td>
<td>.00 (.00)</td>
<td>.00 (.00)</td>
</tr>
<tr>
<td>Frequency of reviews</td>
<td>.03** (.01)</td>
<td>.00* (.00)</td>
</tr>
<tr>
<td>Treatment x Prior Knowledge Interaction</td>
<td>--</td>
<td>.05 (.09)</td>
</tr>
<tr>
<td>Treatment x URM Interaction</td>
<td>--</td>
<td>.10 (.04)</td>
</tr>
<tr>
<td>Random Effects</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Student-Level</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Prior Knowledge Score</td>
<td>.40** (.06)</td>
<td>.37** (.06)</td>
</tr>
<tr>
<td>Pretest Interest Score</td>
<td>.00 (.00)</td>
<td>.00 (.00)</td>
</tr>
<tr>
<td>Pretest Competence Expectancy Score</td>
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<tr>
<td>Classroom-Level</td>
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<td>.0029</td>
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<tr>
<td>Student-Level</td>
<td>.01 (.02)</td>
<td>.01 (.02)</td>
</tr>
<tr>
<td>Proportion Reduction in Variance (Model 1)</td>
<td>--</td>
<td>.0046</td>
</tr>
</tbody>
</table>

Notes: URM = Underrepresented minority; & (SE); *Predictor is grand-mean centered; p < .10, *p < .05, **p < .01; --not included in model

AlgebraByExample improves procedural knowledge

Results: Standardized Test Items

7 point gain in standardized test items

Instructional materials based on knowledge of children’s thinking effectively improves students’ performance on skills tested in standardized tests, and also increases their conceptual understanding and procedural problem solving skills.

Struggling students experience the biggest boost in conceptual understanding
Additional findings from translational studies on worked examples

- Incorrect examples are particularly important for improving conceptual knowledge

Booth, Lange, Koedinger, & Newton (2013), Learning and Instruction

Additional findings from translational studies on worked examples

- Incorrect examples are particularly beneficial for struggling students

Barbieri & Booth (2016), Learning and Individual Differences
Additional findings from translational studies on worked examples

- Worked examples may not be generally beneficial for learning middle school geometry content
- But improve 6th and 7th grade low-spatial students’ ability to learn the content

\[\begin{array}{ccc}
\text{Pretest} & \text{Posttest} & \text{Example} \\
0.75 & 0.8 & 0.85 \\
0.8 & 0.9 & 0.95 \\
\end{array}\]

Booth, Stepnowski, & Bradley (in prep)

- Correct and incorrect worked examples for fraction arithmetic in 6th grade lead to improved fraction magnitude knowledge (but not whole number magnitude knowledge)
- Students who begin with low fraction magnitude knowledge benefit the most

\[\begin{array}{cccc}
\text{Pre-test Whole Number PAE} & \text{Pre-test Overall Fraction PAE} \\
0.600^\text{**} & 0.300^\text{**} \\
(0.040) & (0.063) \\
\end{array}\]

Booth & Barbieri (in prep)
Take-aways:

- Including examples and self-explanation in classroom assignments can lead to marked improvements in student learning.

Effectiveness may vary based on:
- Content area/type of knowledge
- Task characteristics
- Student characteristics
- Interactions between student characteristics/task characteristics/content area

Cognitive Science in Math Teacher Preparation

- Deans for Impact
  - National organization of leaders in educator preparation who are committed to transforming practice and elevating the teaching profession
  - Resource on the best evidence available from research on how learning occurs
6 key questions that every teacher candidate should grapple with and be able to answer

Goals:

- Improve teacher-candidates’ understanding of science of learning principles and their applicability

HOW DO STUDENTS UNDERSTAND NEW IDEAS?

COGNITIVE PRINCIPLES

Students learn new ideas by reference to ideas they already know.

To learn, students must transfer information from working memory (where it is consciously processed) to long-term memory (where it can be stored and later retrieved). Students have limited working memory capacities that can be overwhelmed by tasks that are cognitively too demanding. Understanding new ideas can be improved if students are confronted with too much information at once.

PRACTICAL IMPLICATIONS FOR THE CLASSROOM

A well-sequenced curriculum is important to ensure that students have prior knowledge they need to master new ideas.

Teachers can strategies because they try to make ideas coherent so that students already know. But analogies are effective only if teachers elaborate on them, and direct students attention to the crucial similarities between existing knowledge and what is to be learned.

- How do we translate this information into teacher preparation?

- Exposure to principles?
- Infusion of principles into existing teacher preparation courses?
- New courses bridging between cognitive science and methodology?
- Co-teaching by cognitive scientists and teacher educators throughout the process?

A preliminary effort: Infusion of principles into existing methodology courses

- Lecture interludes and discussion of methodology with SOL lens
- How does this improve preservice teachers’ knowledge and application of SOL principles?
Method

- Participants:
  - 71 juniors in Early Childhood Education program (PreK-4)
  - 2 sections treatment (N = 47)
  - 1 section control (N = 25)

- Procedure:
  - Cognitive science lessons revisited in Math methods course
  - Cognitive Development course in prior semester
  - Encouraged to discuss implications for teaching and lesson-planning

- Measures:
  - Knowledge and Applications test
  - Lesson Plan justifications/reflections

Preliminary Results: Does cognitive science infusion improve knowledge of what influences student learning?
Lesson Plan Reflections: Comparing Math Methods and Cog Dev classes

- **CogSci Reasons:**
  - Prior Knowledge
  - Comparison
  - Encoding
  - Expertise
  - Misconceptions
  - Problem Solving
  - Attention Focusing
  - Working Memory

- **Folk Reasons:**
  - Manipulatives/concrete thinking
  - Constructivist/active learning
  - Learning Styles
  - Modeling
  - Multiple Intelligences
  - Real-world Connections
  - Breaking down steps
Preliminary Results: What do elementary preservice teachers reference when justifying lesson choices?

Mean mentions per student

- Treatment
- Control

Overlapping waves in teacher preparation strategies regarding children’s thinking

- Only knowledge of pedagogy
- Include knowledge of cognitive science (separately)
- Integrated knowledge of cognitive science and pedagogy
Discussion

- Cognitive science can be used to have an impact on real world classrooms
  - Materials and training
- The work to make cognitive principles accessible for the real world is not trivial
  - It is not sufficient to just “apply” knowledge of how children think
    - Thorough literature/cognitive task analysis
    - True collaboration between researchers, content specialists, and practitioners
    - Data from lab and classroom studies
    - Adjust and repeat!
- More of this translational work in Cognitive Science is critical to making an appreciable difference in our education system