Bob Siegler’s Contributions to Mathematics Education Policies and Practices

Diane J. Briars
Immediate Past President
National Council of Teachers of Mathematics
dbriars@nctm.org
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So what?

Working with Bob
NCTM Mission Statement

NCTM is the public voice of mathematics education, supporting teachers to ensure equitable mathematics learning of the highest quality for all students through vision, leadership, professional development, and research.

NCTM Strategic Priorities

• Access and Equity
• Advocacy
• Curriculum, Instruction, and Assessment
• Professional Development
• Research
• Technology
National Assessment of Educational Progress

4th Grade

8th Grade
High School

NAEP Grade 12

Calculus College vs AP Exams

\[ \frac{3}{4} + \frac{1}{3} \]

Phil Daro, 2010
Other “Butterflies”

- FOIL
- Cross multiplication
- Division of fractions: Keep-change-flip, KFC, etc.
- $a - b = a + \bar{b}$: Keep-change-change
- Key words
- Division algorithm: Does McDonalds Sell Cheese Burgers?
“Rules That Expire”

- Addition and multiplication give results larger than the original numbers.
- Subtraction and division give results smaller than at least one of the numbers.
- The longer the number, the larger the number.
- When you multiply a number by ten, just add a zero to the end of the number.
- You always divide the larger number by the smaller number.

Karp, Bush & Dougherty, 2014

Our Challenge

Shift from emphasis on: How to get answers

To emphasis on: Understanding mathematics
Principles to Actions: Ensuring Mathematical Success for All

- Describes the **supportive conditions, structures, and policies** required to give all students the power of mathematics
- Focuses on **teaching and learning**
- Emphasizes engaging students in **mathematical thinking**
- Describes how to ensure that mathematics achievement is maximized for **every student**
- Is not specific to any standards; it’s **universal**

**Principle on Teaching and Learning**

An excellent mathematics program requires effective teaching that engages students in meaningful learning through individual and collaborative experiences that promote their ability to make sense of mathematical ideas and reason mathematically.
Effective Mathematics Teaching Practices

1. Establish mathematics **goals** to focus learning.
2. Implement **tasks** that promote reasoning and problem solving.
3. Use and connect mathematical **representations**.
4. Facilitate meaningful mathematical **discourse**.
5. Pose purposeful **questions**.
6. Build **procedural fluency** from conceptual understanding.
7. Support **productive struggle** in learning mathematics.
8. **Elicit and use evidence** of student thinking.

A candy jar contains 5 Jolly Ranchers (squares) and 13 Jawbreakers (circles). Suppose you had a new candy jar with the same ratio of Jolly Ranchers to Jawbreakers, but it contained 100 Jolly Ranchers. How many Jawbreakers would you have? Explain how you know.

<table>
<thead>
<tr>
<th>Group 1 (incorrect, additive)</th>
<th>Groups 3 and 5 (scale factor)</th>
<th>Groups 4 and 7 (scaling up)</th>
</tr>
</thead>
<tbody>
<tr>
<td>106 JRs is 93 more than the 5 we started with. So we will need 93 more JBs than the 13 we started with.</td>
<td>You had to multiply the five JRs by 20 to get 100, so you’d also have to multiply the 13 JBs by 20 to get 260.</td>
<td>JR</td>
</tr>
<tr>
<td>5 JRs = 95 JBs = 100 JRs</td>
<td>(x 20)</td>
<td>5 JRs</td>
</tr>
<tr>
<td>13 JBs + 95 JBs = 108 JBs</td>
<td>(x 20)</td>
<td>13 JBs</td>
</tr>
<tr>
<td>So then you just multiply 2.6 by 100.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 JR</td>
<td>100 JRs</td>
<td></td>
</tr>
<tr>
<td>2.6 JBs</td>
<td>260 JBs</td>
<td></td>
</tr>
</tbody>
</table>

We started by doubling both the number of JRs and JBs. But then when we got to 80 JRs, we didn’t want to double it anymore because we wanted to end up at 100 JRs, and doubling 80 would give us too many. So we noticed that if we added 20 JRs, 52 JBs, and 80 JRs: 100 JRs; we would get 100 JRs: 260 JBs.

Group 2 (unit rate)

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We drew 100 JRs in groups of 3. Then we put 11 JBs with each group of 3 JRs. We then counted the number of JBs and found we had used 260 of them.
Implications of Cognitive Science Research for Mathematics Education

- Mathematical understanding before children enter school
- Pitfalls in mathematics learning
- Cognitive variability and strategy choice
- Individual differences
- Discovery and insight
- Relationship between conceptual and procedural knowledge
- Cooperative learning
- Promoting analytic thinking and transfer

Siegler, 2003
Implications of Cognitive Science Research for Mathematics Education

“Allowing children to use the varied strategies that they generate, and helping them understand why superficially-different strategies converge on the right answer and why superficially reasonable strategies are incorrect seems likely to build deeper understanding.”

Siegler, 2003

Analytic thinking as a central goal--

• A set of processes for identifying the causes of events.
• Inherently constructive—demands children actively think about causes of events.
• Involves purposeful engagement
• Promotes transfer
• Encourage by asking students to explain their conclusions or answers of others.

Siegler, 2003
Adding It Up: Helping Children Learn Mathematics

Strands of Mathematical Proficiency

- **Conceptual Understanding** – comprehension of mathematical concepts, operations, and relations
- **Procedural Fluency** – skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- **Strategic Competence** – ability to formulate, represent, and solve mathematical problems
- **Adaptive Reasoning** – capacity for logical thought, reflection, explanation, and justification
- **Productive Disposition** – habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy.


National Research Council, 2001
Common Core State Standards
Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

National Mathematics Advisory Panel

President Bush created in April 2006.
Charge: Rely upon the “best available scientific evidence” and recommend ways “...to foster greater knowledge of and improved performance in mathematics among American students.”
National Mathematics Advisory Panel

• What is the essential content of school algebra and what do children need to know before starting to study it?
• What is known from research about how children learn mathematics?
• What is known about the effectiveness of instructional practices and materials?
• How can we best recruit, prepare, and retain effective teachers of mathematics?
• How can we make assessments of mathematical knowledge more accurate and more useful?
• What do practicing teachers of algebra say about the preparation of students whom they receive into their classrooms and about other relevant matters?
• What are the appropriate standards of evidence for the Panel to use in drawing conclusions from the research base?

Bob’s Contributions to the NMAP
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• Being an internationally respected cognitive scientist who has spent much of his career in issues/practice/research related to learning and mathematics.

• Bob’s willingness to listen and, as needed, compromise allowed several issues related to the work of the Panel to move forward.

NMAP recommendations that have been enacted in CCSS-M

• Streamline the mathematics curriculum in Grades PreK–8 and emphasize a well-defined set of the most critical topics in the early grades.

• Use research about how children learn, especially by recognizing:
  a) the advantages for children in having a strong start;
  b) the mutually reinforcing benefits of conceptual understanding, procedural fluency, and automatic (i.e., quick and effortless) recall of facts; and
  c) that effort, not just inherent talent, counts in mathematical achievement.
<table>
<thead>
<tr>
<th></th>
<th>Numbers and Operations in Base Ten</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Use place value understanding and properties of operations to add and subtract.</td>
</tr>
<tr>
<td>2</td>
<td>Use place value understanding and properties of operations to add and subtract.</td>
</tr>
</tbody>
</table>
| 3 | Use place value understanding and properties of operations to perform multi-digit arithmetic.  
   *A range of algorithms may be used.* |
| 4 | Use place value understanding and properties of operations to perform multi-digit arithmetic.  
   *Fluently add and subtract multi-digit whole numbers using the standard algorithm.* |
| 5 | Perform operations with multi-digit whole numbers and with decimals to hundredths.  
   *Fluently multiply multi-digit whole numbers using the standard algorithm.* |
| 6 | Compute fluently with multi-digit numbers and find common factors and multiples.  
   *Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.* |
NMAP recommendations that have been enacted in CCSS-M

- A major goal for K–8 mathematics education should be proficiency with fractions (including decimals, percent, and negative fractions). . . . a conceptual understanding of fractions and decimals and the operational procedures for using them are mutually reinforcing. One key mechanism linking conceptual and procedural knowledge is the ability to represent fractions on a number line.
- Teachers’ regular use of formative assessment improves their students’ learning.

IES Practice Guide

Offers educators specific evidence-based recommendations that address the challenge of improving students’ understanding of fraction concepts in kindergarten through 8th grade.
Recommendation 1.

Build on students’ informal understanding of sharing and proportionality to develop initial fraction concepts.

- Use equal-sharing activities to introduce the concept of fractions. Use sharing activities that involve dividing sets of objects as well as single whole objects.
- Extend equal-sharing activities to develop students’ understanding of ordering and equivalence of fractions.
- Build on students’ informal understanding to develop more advanced understanding of proportional reasoning concepts. Begin with activities that involve similar proportions, and progress to activities that involve ordering different proportions.
Bob’s Leadership of the IES Panel

Bob was a great moderator during spirited debates. He had a way of helping people come to the conclusion that they were both probably advocating for similar positions, just in their own unique ways. He was the master of finding middle ground and/or compromise. It was a pleasure to serve under his leadership.

Jon Wray
Early Predictors of High School Mathematics Achievement

Robert S. Siegler¹, Greg J. Duncan², Pamela E. Davis-Kean³,⁴, Kathryn Duckworth⁵, Amy Claessens⁶, Mimi Engel⁷, Maria Ines Susperreguy⁸,⁹, and Meichu Chen¹

¹Department of Psychology, Carnegie Mellon University; ²Department of Education, University of California, Irvine; ³Department of Psychology, University of Michigan; ⁴Institute for Social Research, University of Michigan; ⁵Quantitative Science, Institute of Education, University of London; ⁶Department of Public Policy, University of Chicago; and ⁷Department of Public Policy and Education, Vanderbilt University

Abstract

Identifying the types of mathematics content knowledge that are most predictive of students’ long-term learning is essential for improving both theories of mathematical development and mathematics education. To identify these types of knowledge, we examined long-term predictors of high school students’ knowledge of algebra and overall mathematics achievement. Analyses of large, nationally representative, longitudinal data sets from the United States and the United Kingdom revealed that elementary school students’ knowledge of fractions and of division uniquely predicts those students’ knowledge of algebra and overall mathematics achievement in high school, 5 or 6 years later, even after statistically controlling for other types of mathematical knowledge, general intellectual ability, working memory, and family income and education. Implications of these findings for understanding and improving mathematics learning are discussed.
The Perils of Analyzing Data Independent of Instructional Experiences

- Teaching practice and opportunity to learn
- Curriculum and instructional materials
Most Significant Contribution

Jim and I have now been together 25 years!

Thank You, Bob!

Mentor
Esteemed Colleague
Dear Friend